Research Institute for Applied Mechanics
Workshop of Oceanographic Radar
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Tsunami Waveform Inversion based on Oceanographic Radar Data

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Today's Topic

Tsunami Waveform Inversion

→Which is … to guess Tsunami Initial Sea Surface Elevation (SSE)
by using Oceanographic Radar Data

>>>Background

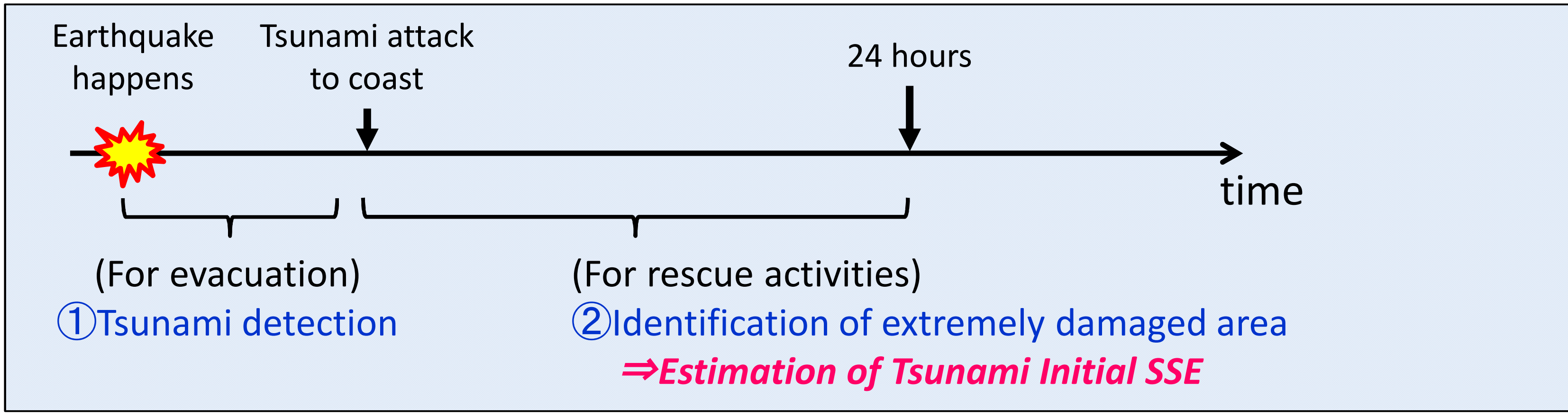
We had 2 problems on 2011 Japan Tsunami…

① Delay of resident's evacuation
② Delay of rescue activities by governments

Causes ① First guess of Tsunami warning was underestimated
② Government offices of themselves were largely damaged,
took much time to identify extremely damaged area

Aim of Research

Time duration



We start research from second problem…

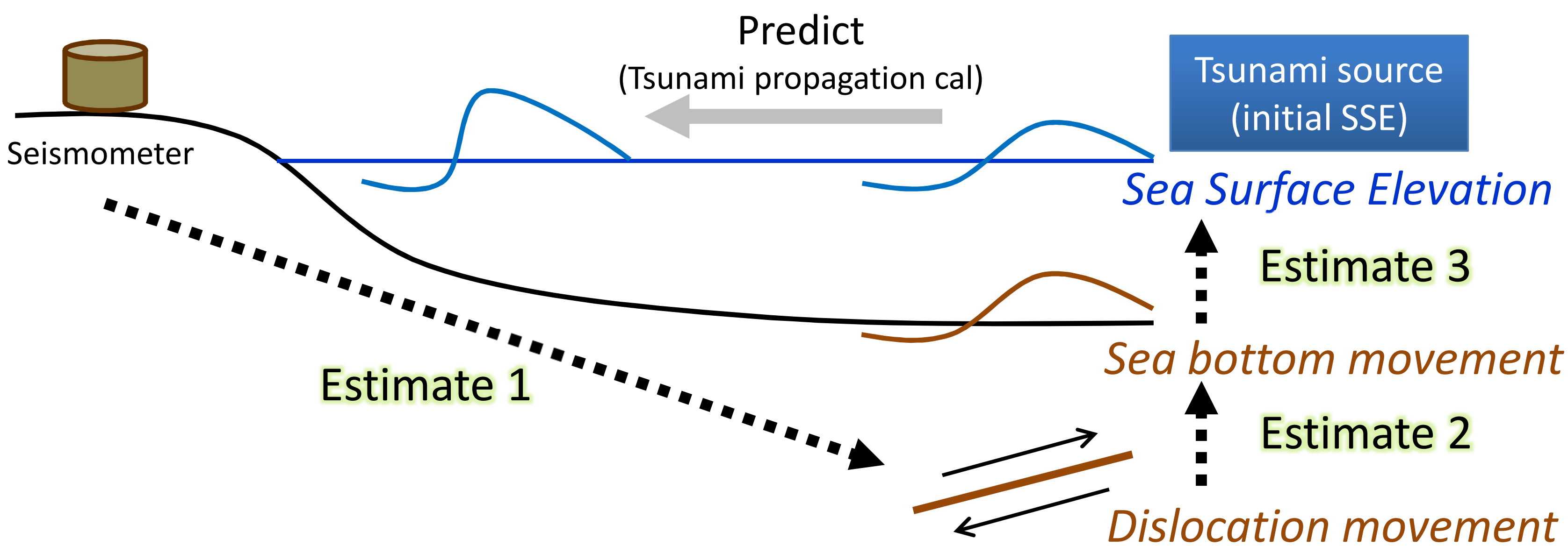
→We try to identify extremely damaged area within 24 hours just after earthquake happens

For that reason…

- *We aim to get Tsunami source information (initial SSE) much accurately by inversion technique*

Current Tsunami Warning System in Japan

- Estimate initial SSE (Sea Surface Elevation) from seismic wave information, not observed Tsunami wave itself



>>> Technical Issues <<<

- It is well known that Tsunami prediction may be underestimated especially for huge earthquake, Tsunami earthquake in case Tsunami initial SSE is disproportioned

Previous Studies

Tsunami Inversion (with GPS or Coastal wave gauge data)

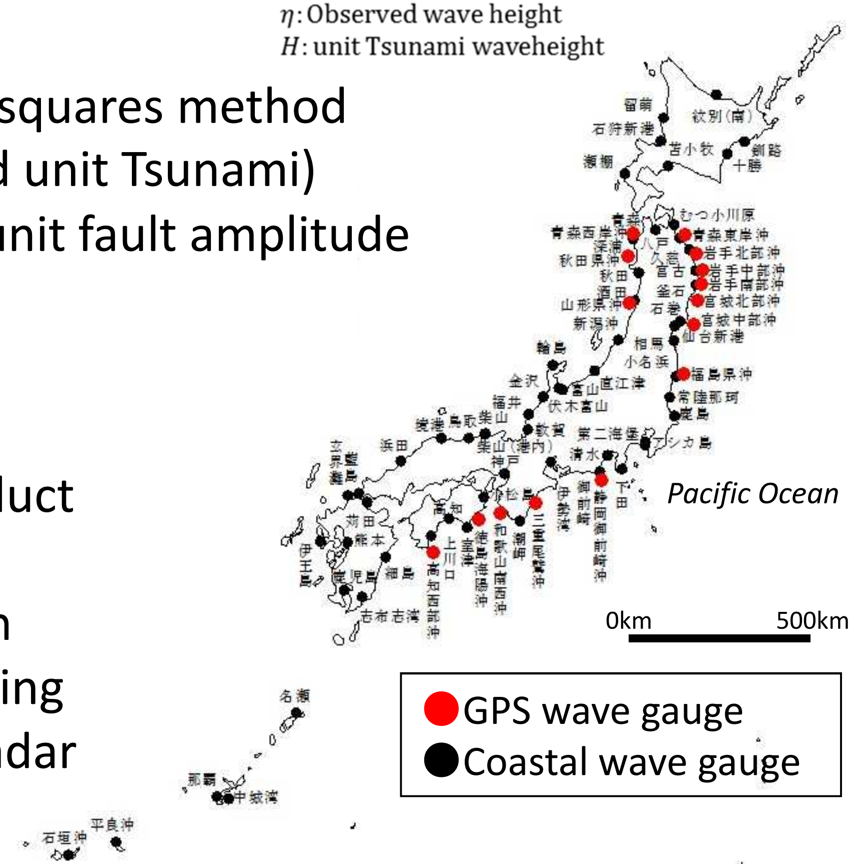
(e.g. Yasuda et al. (JSCE, 2006, 2007), Tatsumi et al. (JSCE, 2008, 2009))

- We use time series of wave height data observed by GPS wave gauge or Coastal wave gauge
- Applying linear long wave theory to Tsunami
  ①Calculate unit Tsunami
  ②Calculate unit fault amplitude by least-squares method
  (minimize error between observed and unit Tsunami)
  ③ Tsunami Initial SSE is given as Sum of unit fault amplitude

$$E = \|\eta - Ha\|^2$$

η: Observed wave height
H: unit Tsunami waveheight

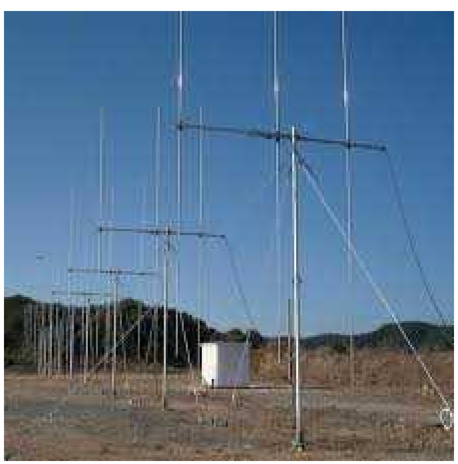
- Disadvantage of GPS or CGW
  →We can know only point information
  →Installed number is not enough to conduct for Tsunami inversion
  →We can get only near shore information
  →As for GPS wave gauge, initial and running costs are much higher than those for radar

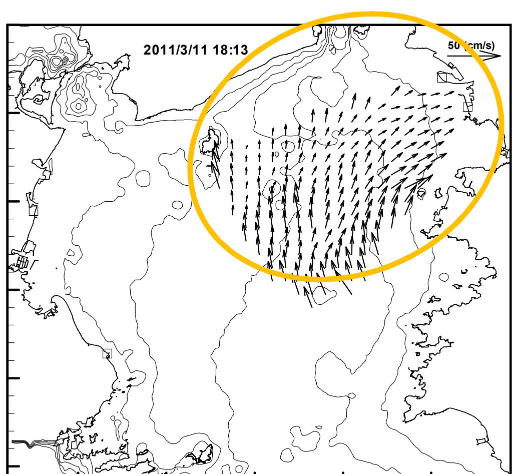


Our Research

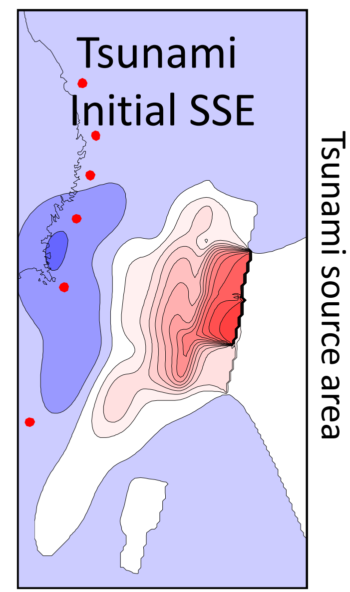
Tsunami Inversion (with observed Radar radial velocity)

- Tsunami current velocities induced by the 2011 Japan Tsunami have been successfully observed by Radars
  (e.g. Hinata et al. (ECSS, 2011), Lipa et al. (Remote Sensing, 2011), Helzel (http://www.helzel.com/))
- Spatial resolution of Oceanographic Radar is much better compared to those of GPS wave gauge or CWG
  → great advantage for Tsunami Inversion
- We are trying to develop Tsunami Inversion method using Radar radial velocity





Inversion by using Radar



Procedure of Inversion

- Applying linear long wave theory to Tsunami motion
- Calculate unit fault amplitude by least squares method
- Tsunami Initial SSE is given as Sum of unit fault amplitude

*In case, unit response function is water surface

①: $\tilde{\eta}$ time, fault unit No.

②: η time

(1) *Divide entire Tsunami source area into some unit fault pieces*

(2) *Give 1m water surface rise to each unit faults*

①: Unit Tsunami calculation data

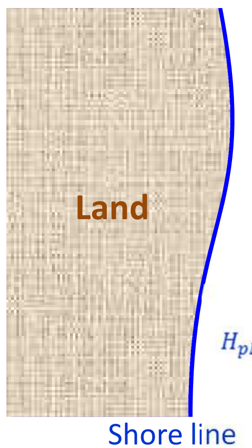
$$H_{p1} = \begin{pmatrix} \tilde{\eta}_{1,1} & \cdots & \tilde{\eta}_{1,N} \\ \vdots & \ddots & \vdots \\ \tilde{\eta}_{T,1} & \cdots & \tilde{\eta}_{T,N} \end{pmatrix}$$

p1

②: Observed data

$$\eta_{p1} = \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_T \end{pmatrix}$$

...



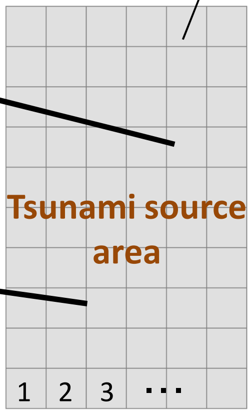
$$H_{pN} = \begin{pmatrix} \tilde{\eta}_{1,1} & \cdots & \tilde{\eta}_{1,N} \\ \vdots & \ddots & \vdots \\ \tilde{\eta}_{T,1} & \cdots & \tilde{\eta}_{T,N} \end{pmatrix}$$

$$\eta_{pN} = \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_T \end{pmatrix}$$

pN

(3) *Carry out unit Tsunami calculation*

(Get time series of water surface at observation sites, p1, p2 … pN)



(4) *Carry out Inversion (least-squares method)*

Unit Tsunami calculation		Observation
$H = \begin{pmatrix} \tilde{\eta}_{1,1} & \cdots & \tilde{\eta}_{1,N} \\ \vdots & \ddots & \vdots \\ \tilde{\eta}_{T,1} & \cdots & \tilde{\eta}_{T,N} \end{pmatrix}$	⟵ Inversion analysis (least-squares method) ⟶	$\eta = \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_T \end{pmatrix}$

Coefficient a (Amplitude)

(5) *Finally, get Tsunami initial SSE*

■ $minimize\ error\ (sum\ of\ squares)\quad \mathrm{E} = \|\eta - Ha\|^2\ \ min \ \rightarrow \frac{\partial E}{\partial a} = 0$

$$H^T Ha = H^T \eta \quad : N\ dimension\ matrix$$

$$\begin{pmatrix} \tilde{\eta}_{1,1} & \cdots & \tilde{\eta}_{1,T} \\ \vdots & \ddots & \vdots \\ \tilde{\eta}_{N,1} & \cdots & \tilde{\eta}_{N,T} \end{pmatrix} \begin{pmatrix} \tilde{\eta}_{1,1} & \cdots & \tilde{\eta}_{1,N} \\ \vdots & \ddots & \vdots \\ \tilde{\eta}_{T,1} & \cdots & \tilde{\eta}_{T,N} \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ a_N \end{pmatrix} = \begin{pmatrix} \tilde{\eta}_{1,1} & \cdots & \tilde{\eta}_{1,T} \\ \vdots & \ddots & \vdots \\ \tilde{\eta}_{N,1} & \cdots & \tilde{\eta}_{N,T} \end{pmatrix} \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_T \end{pmatrix}$$

→ a gives Tsunami initial SSE

Numerical Experiment

(2011 Japan Tsunami)

Numerical Experiment, Tohoku Area Japan

We carried out 2 cases of inversion, using data are as follows:

- Case① : Radar 2sites, radial velocity, sampling interval : **10sec**
- Case② : Radar 2sites, radial velocity, **2min** average, sampling interval : **1min**

corresponding to practical Radar observation

STEP 1

Calculation of unit Tsunami (760 unit faults)

→Output data (stocked) at Radar site : Radial velocity

→Government equation : linear long wave equation

$$\frac{\partial \eta}{\partial t}+\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}=0 \qquad \frac{\partial M}{\partial t}+gD\frac{\partial \eta}{\partial x}=0 \qquad \frac{\partial N}{\partial t}+gD\frac{\partial \eta}{\partial y}=0$$

→Spatial resolution : 1 km

→Calculation time step : 2 sec

→Unit fault size : 15 km × 15km

→Tsunami source area : 300 km × 600 km (760 unit faults)

STEP 2

Making of observation data (Velocity)

→Calculate Tsunami initial SSE applying Fujii Satake ver4.2 fault model

→Calculate Tsunami propagation applying estimated Tsunami initial SSE

→Calculation conditions and output data are the same as STEP 1

STEP 3

Tsunami wave form inversion

→Calculate Tsunami initial SSE by least-squares method

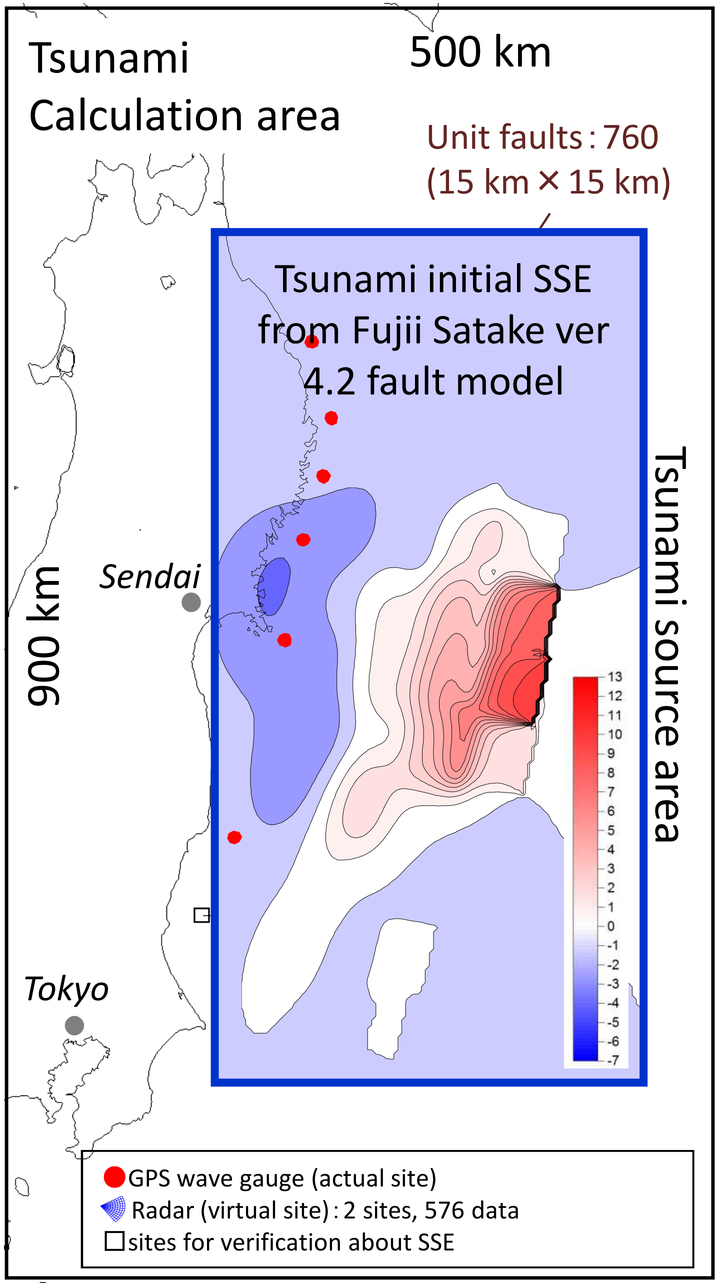
→Data sampling interval : 2 hour (for identify extremely damaged area)

STEP 4

Validation (Error estimation)

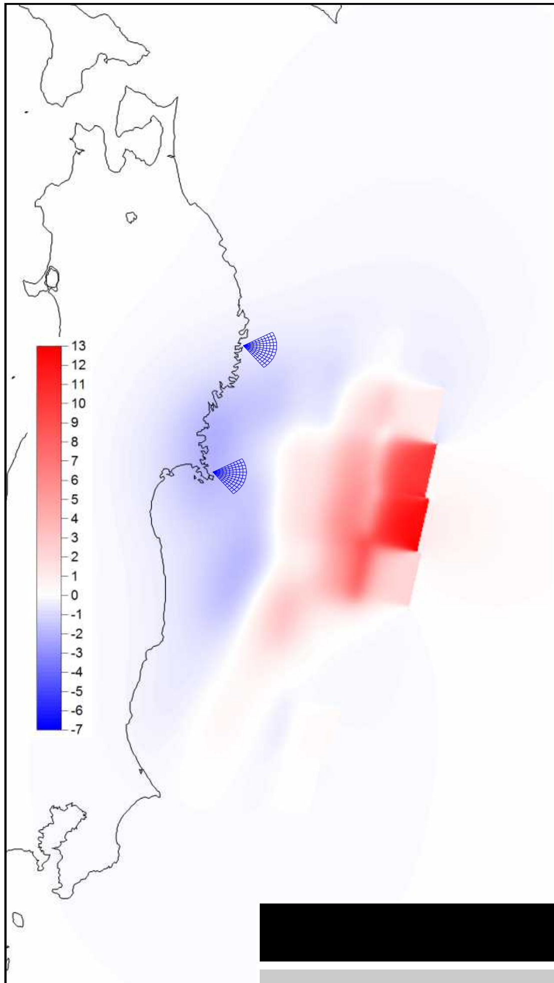
→Calculate Tsunami propagation for 6 hours applying estimated Tsunami initial SSE in STEP 3

→Compare calculated SSE (STEP3 result and STEP 2 result) at the sites (●□)

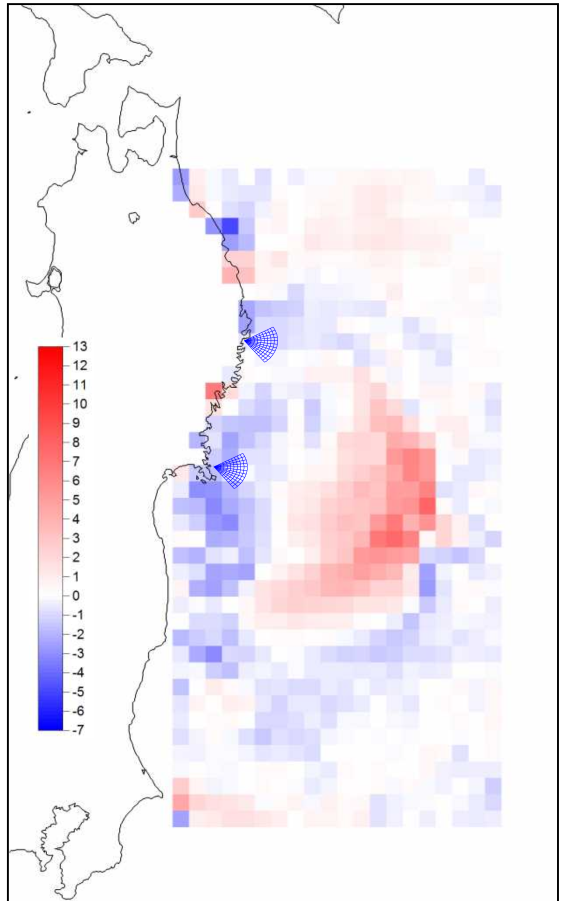


Distribution of Tsunami Initial SSE

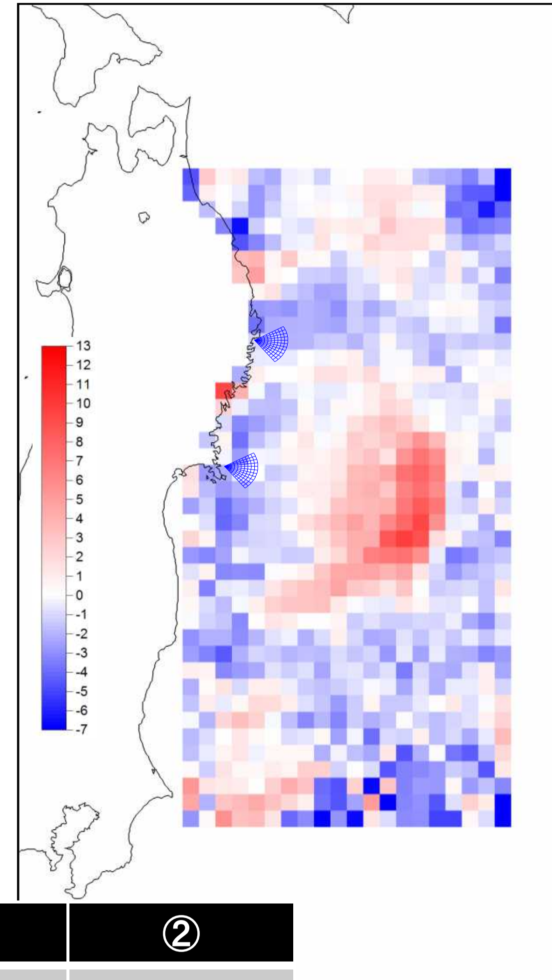
True initial SSE
(Fujii Satake ver4.2 fault model)



①Radar Radial Velocity
(sampling interval : **10 sec**)

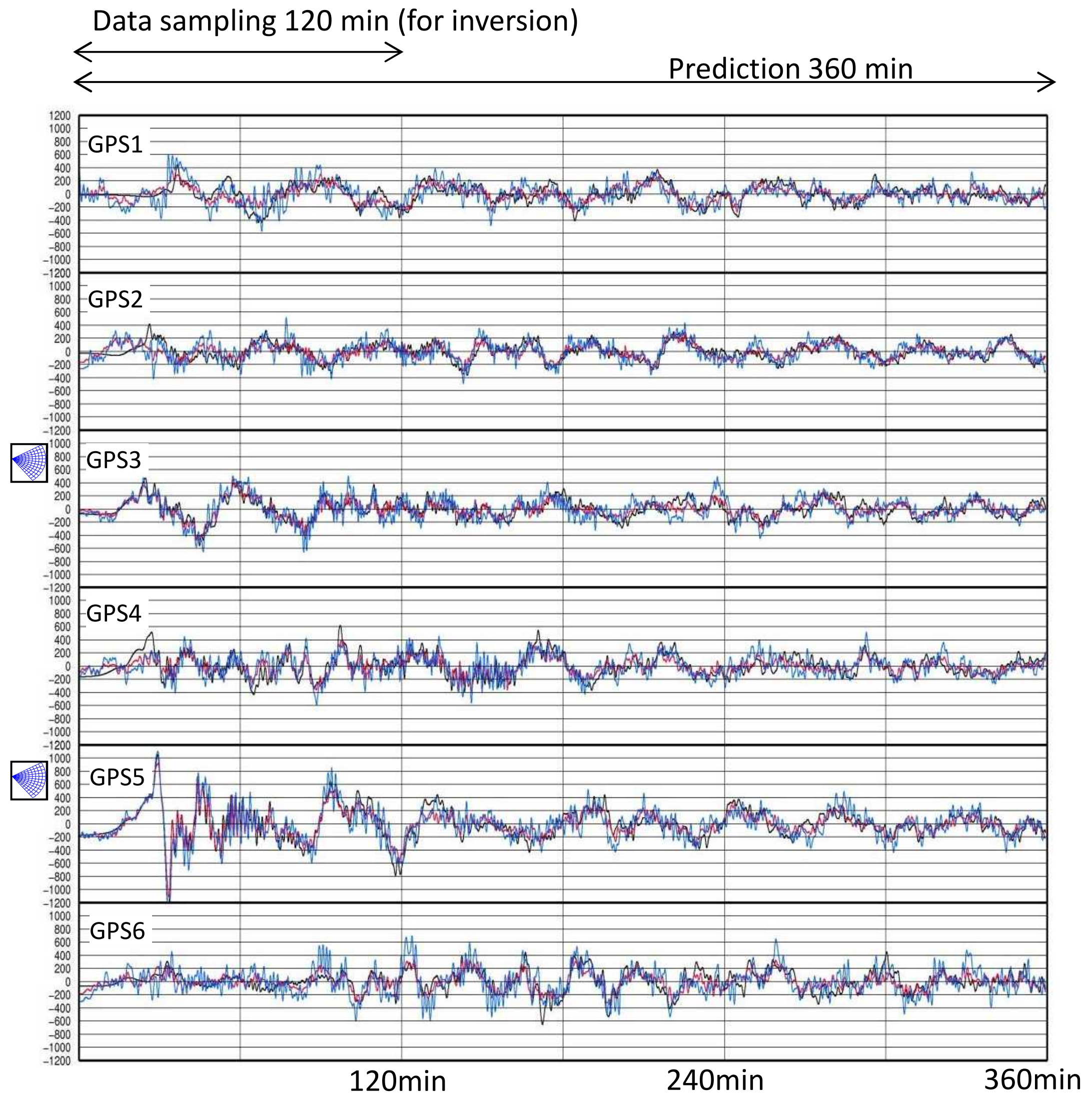


②Radar Radial Velocity
(**2 min** average,
sampling interval : **1 min**)

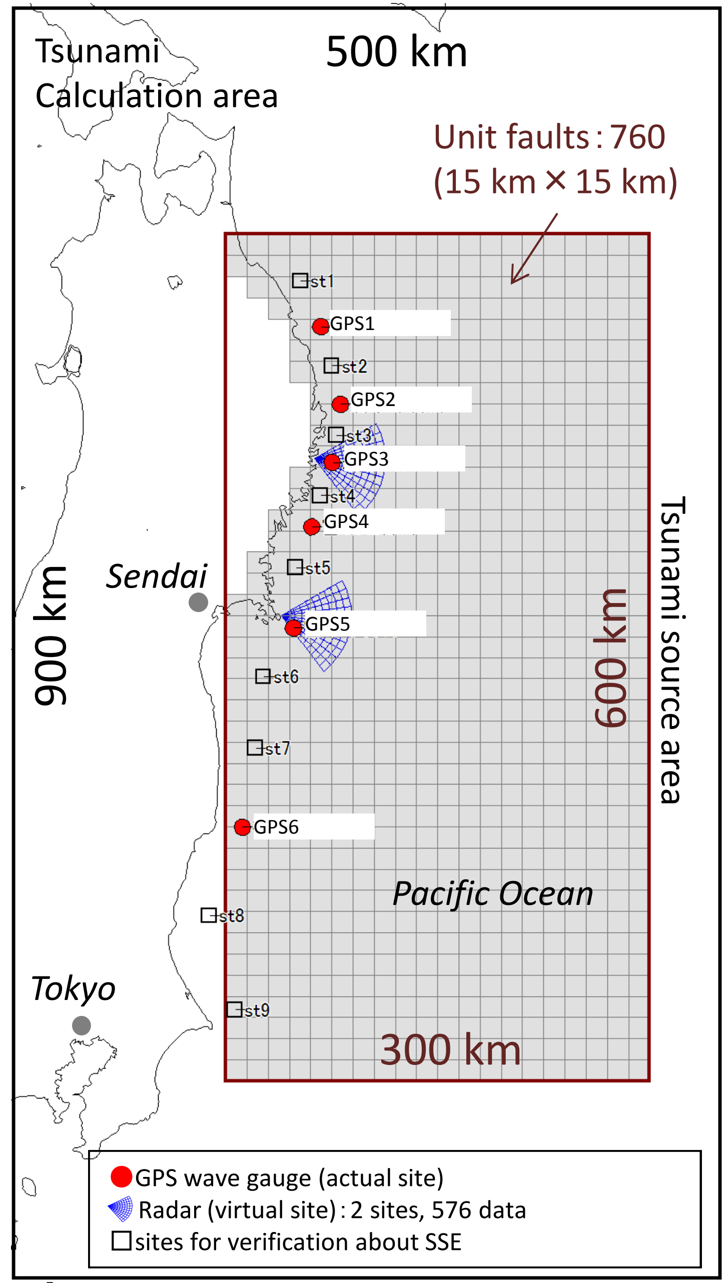


RMS error	①	②
RMSE (m)	0.67	0.86
RMSE／maximum wave height (True value) (%)	5.3	6.8

Time Series of Wave Height



— Radar Radial Velocity (sampling interval : **10 sec**)
— Radar Radial Velocity (**2 min** average, sampling interval : **1 min**)
— True Wave Height



Time Series of Wave Height

➤ Variance Reduction*

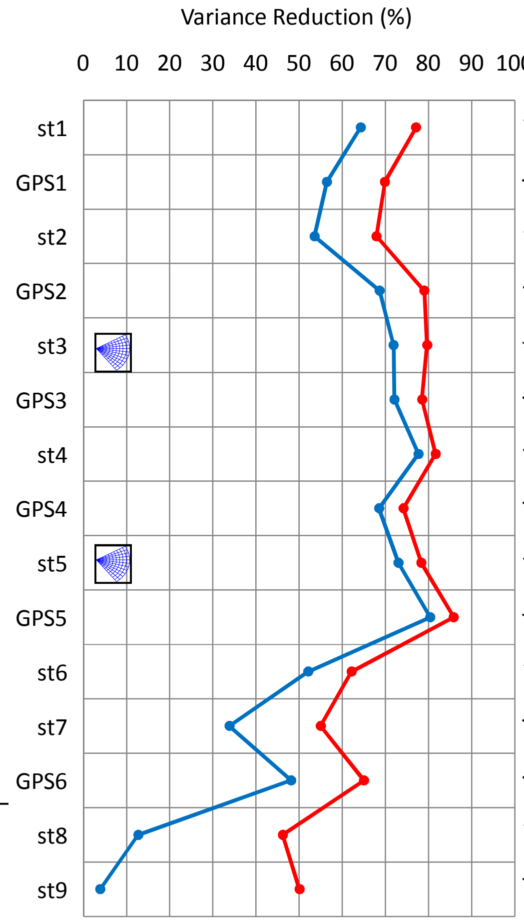
This value shows agreement (concordance rate) of wave height between observation and prediction

Variance Reduction (%)

$$VR = \left(1 - \frac{\sum_{j=1}^{T}(\eta_j - \hat{\eta}_j)^2}{\sum_{j=1}^{T}\eta_j^{\,2}}\right) \times 100$$

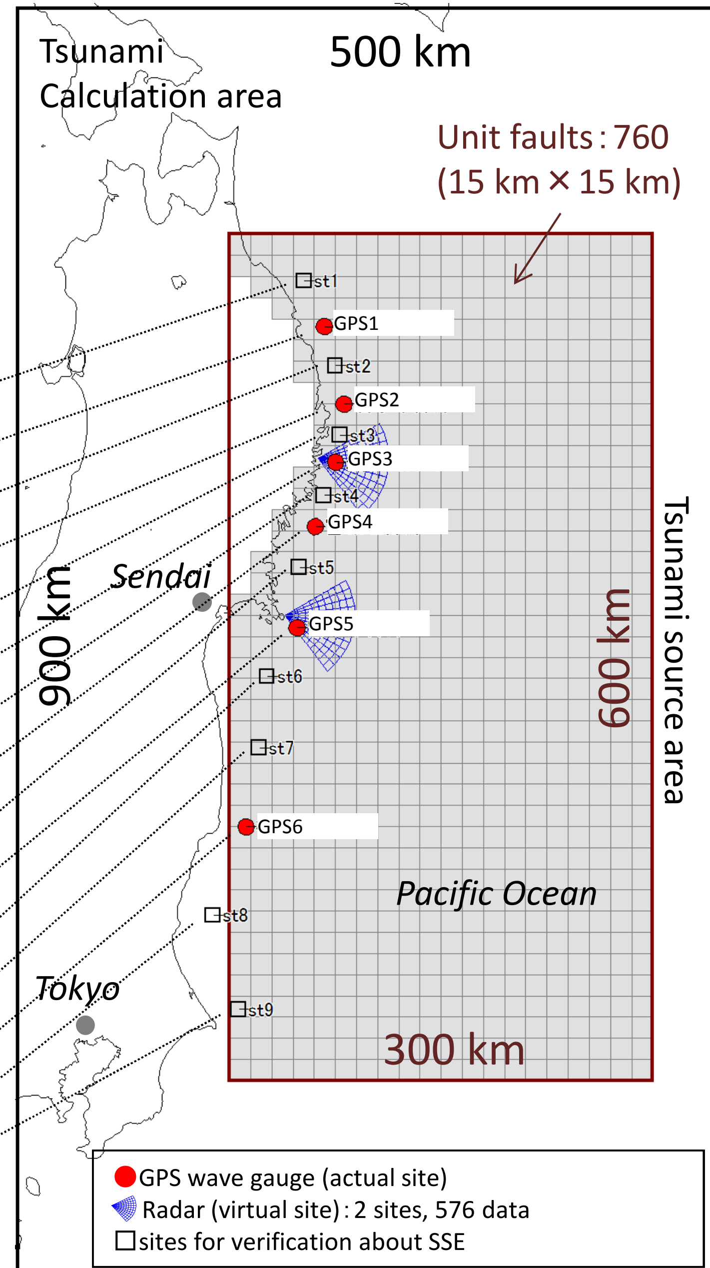
η_j : Prediction by inversion
$\hat{\eta}_j$: Observation (true value)

*NIED (2008) :
As for seismic wave, this value is used to assess quality of seismic moment tensor solution



— Radar Radial Velocity (sampling interval : **10 sec**)
— Radar Radial Velocity (**2 min** average, sampling interval : **1 min**)

(VR for 6 hours)



Questions ?

We would like to know …

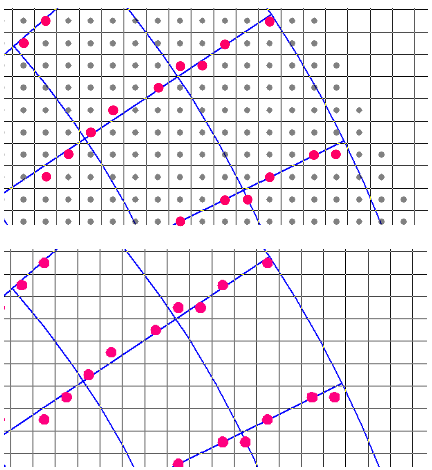
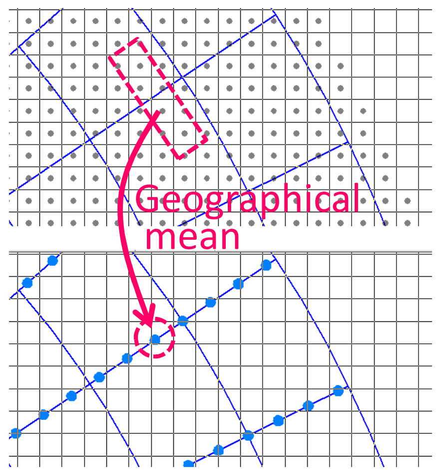
- How we decide optimum number and location of Oceanographic Radar for estimation of Tsunami initial SSE
- What is the required specification of Radar for Tsunami inversion?

→*Now we are trying on to answer these questions thorough idealized experiments*

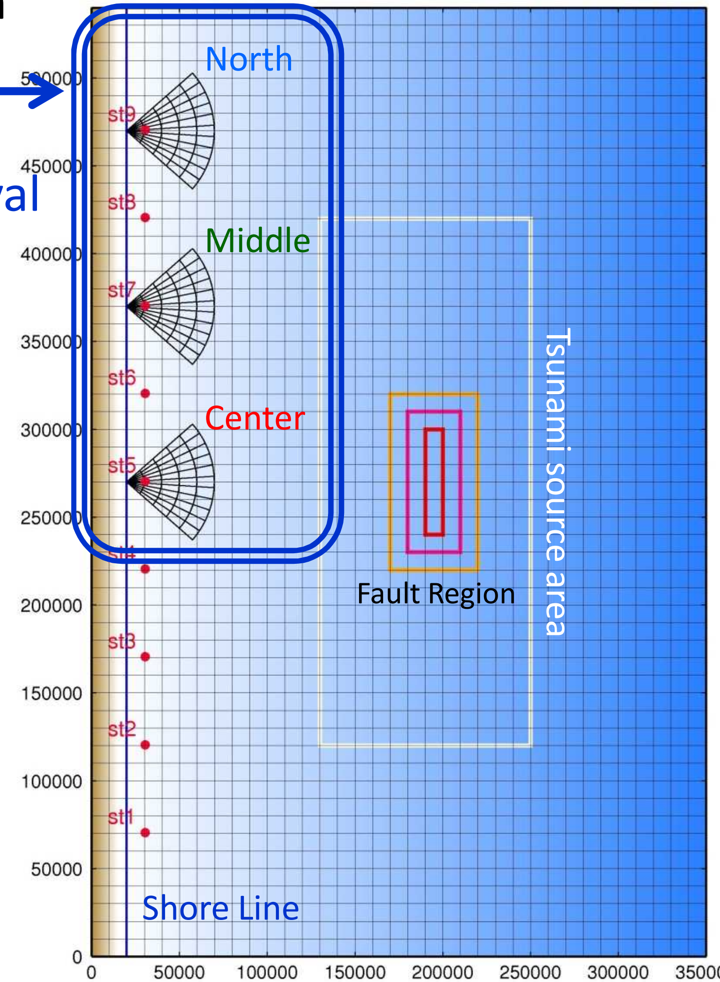
Aim of Experiment

We would like to know about …

- Effect of these conditions on Tsunami Inversion
  - (1) Location of Radar site
  - (2) Spatial resolution of Radar velocity
  - (3) Temporal average and data sampling interval

No Average	Spatial Average
Tsunami calculation mesh  points for Inversion	Tsunami calculation mesh  points for Inversion

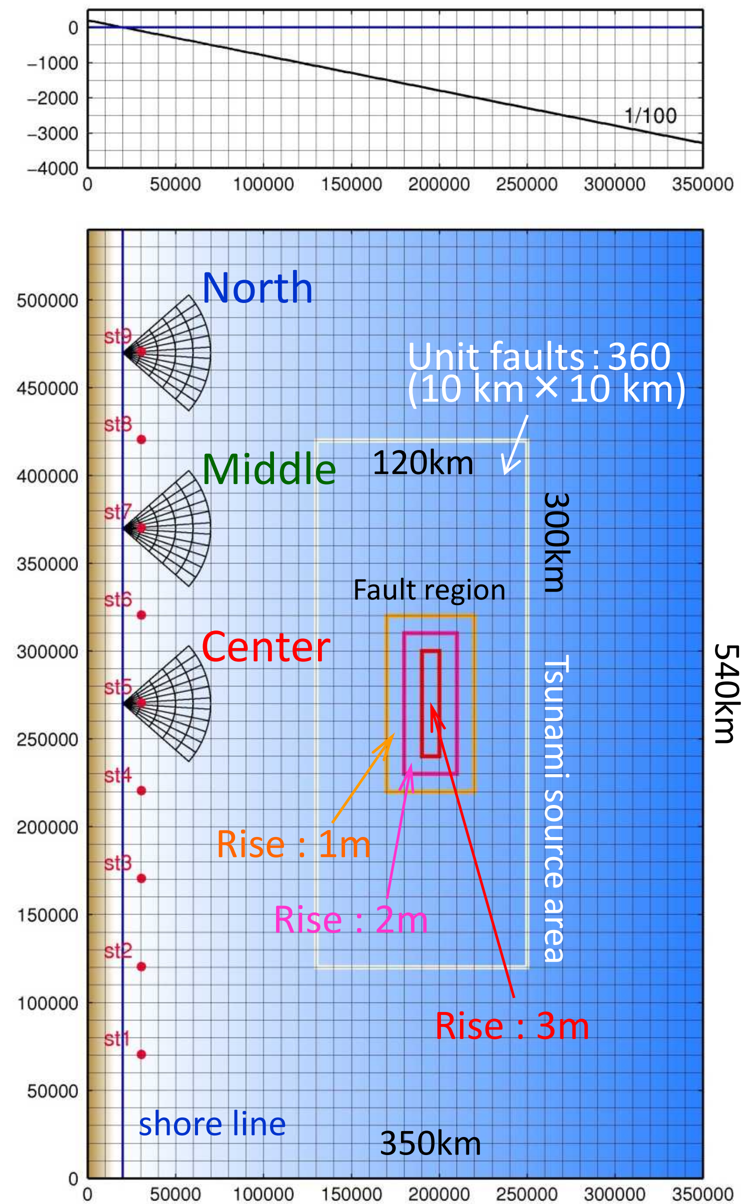
No Average	Temporary Average
--- Sampling interval 10 sec	2 min moving average Sampling interval 1 min



Corresponding to practical Radar

Experiment Conditions

- Geometry : 1/100 bottom slope
- Fault Model rise : 3 m, 2 m, 1 m
- Tsunami calculation
  →spatial resolution : 1 km
  →time step : 2 sec
  →unit fault size : 10 km × 10 km
  →calculation period : 2 hour
- Radar location : North, Middle, Center
- Inversion conditions
  →data sampling period : 40min, 60min
  →using data : Radar radial velocity



Inversion Result

influences of

Radar Location

Inversion Result ① (Radar Location)

(1) Radar location : **Center**

(2) Spatial average : No

(3) Sampling interval : 10 sec

➢ Data sampling period : 40min

Time first wave reach coast

st8

st7

st6

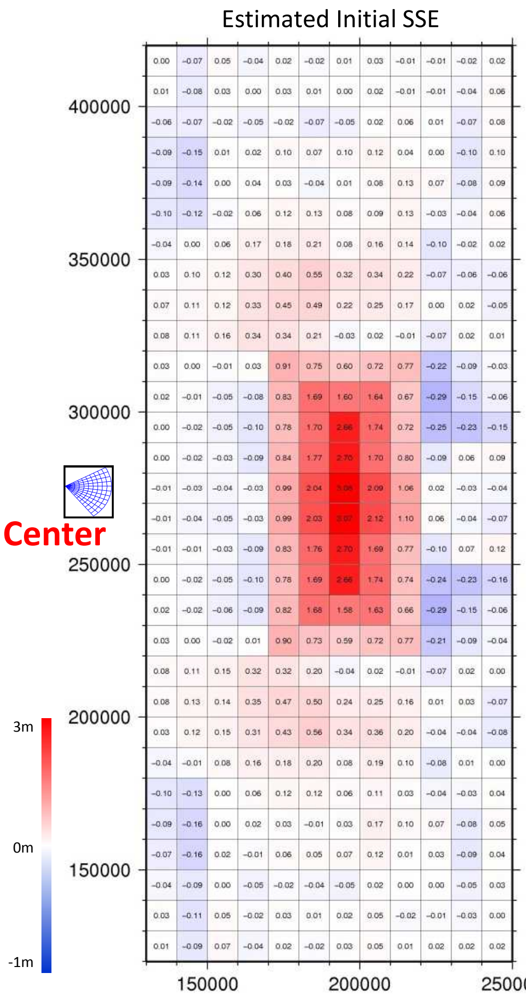
st5

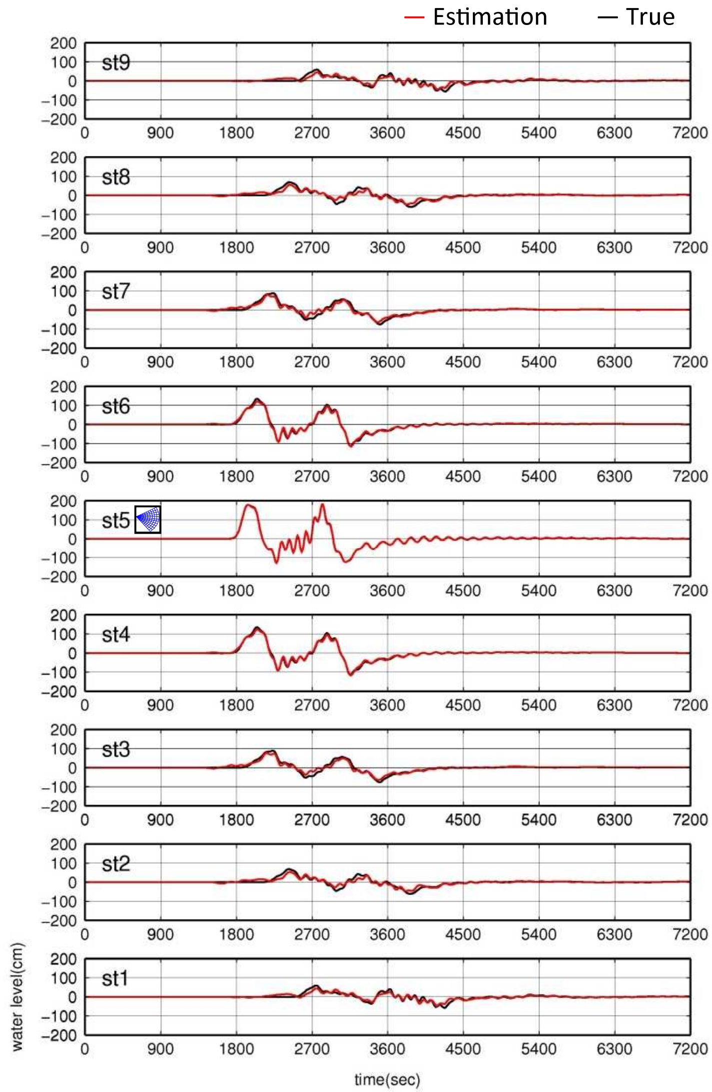
Center

st4

st3

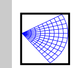
st2





RMSE (m)
0.069

Variance Reduction (%)	(6 hours)
St9	81.1
St8	83.4
St7	90.8
St6	98.9
St5	99.9
St4	99.0
St3	91.8
St2	83.8
St1	81.8



Center

Inversion Result ② (Radar Location)

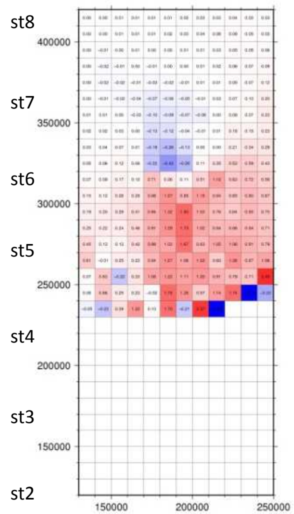
(1) Radar location : **Middle**, **North**

(2) Spatial average : No

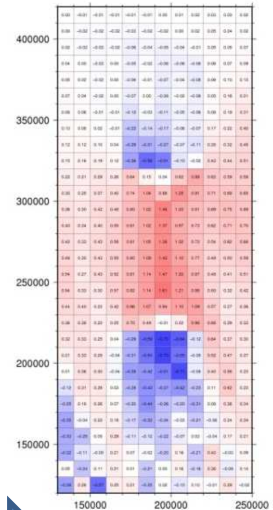
(3) Sampling interval : 10 sec

➢ Data sampling period : 40min, 60min

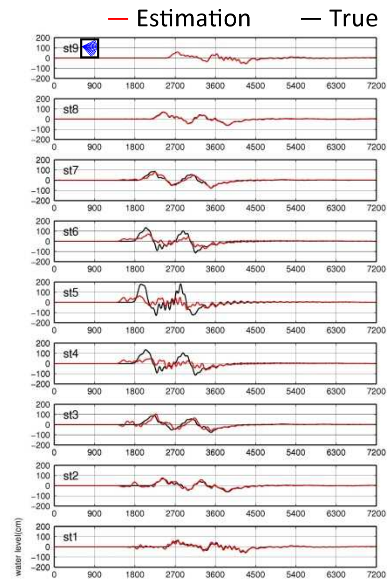
North



40min



60min



RMSE (m)	
0.185	
Variance Reduction (%) (6 hours)	
St9	99.5
St8	98.7
St7	86.2
St6	60.2
St5	26.1
St4	34.5
St3	68.2
St2	85.4
St1	86.1

good

worse

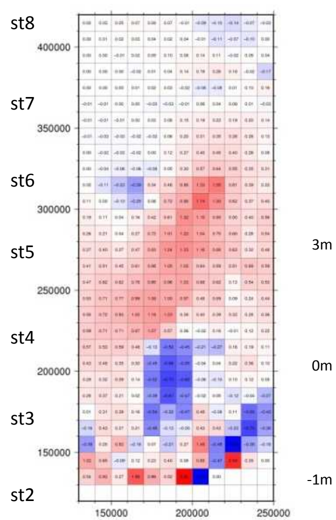
good

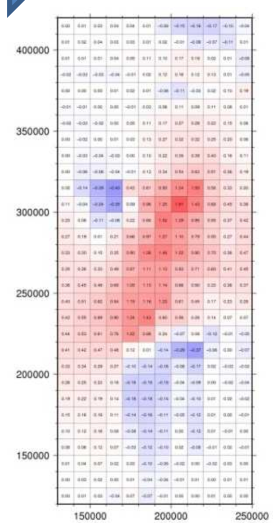
North

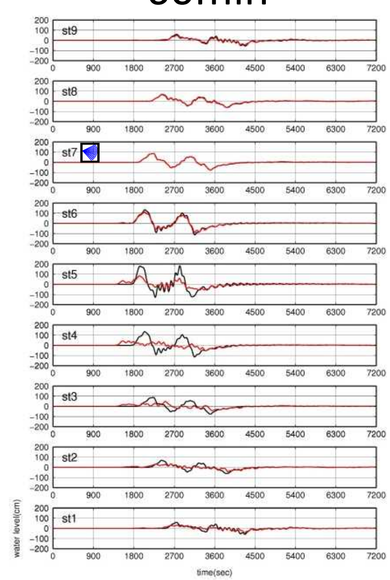
60min

60min

Middle



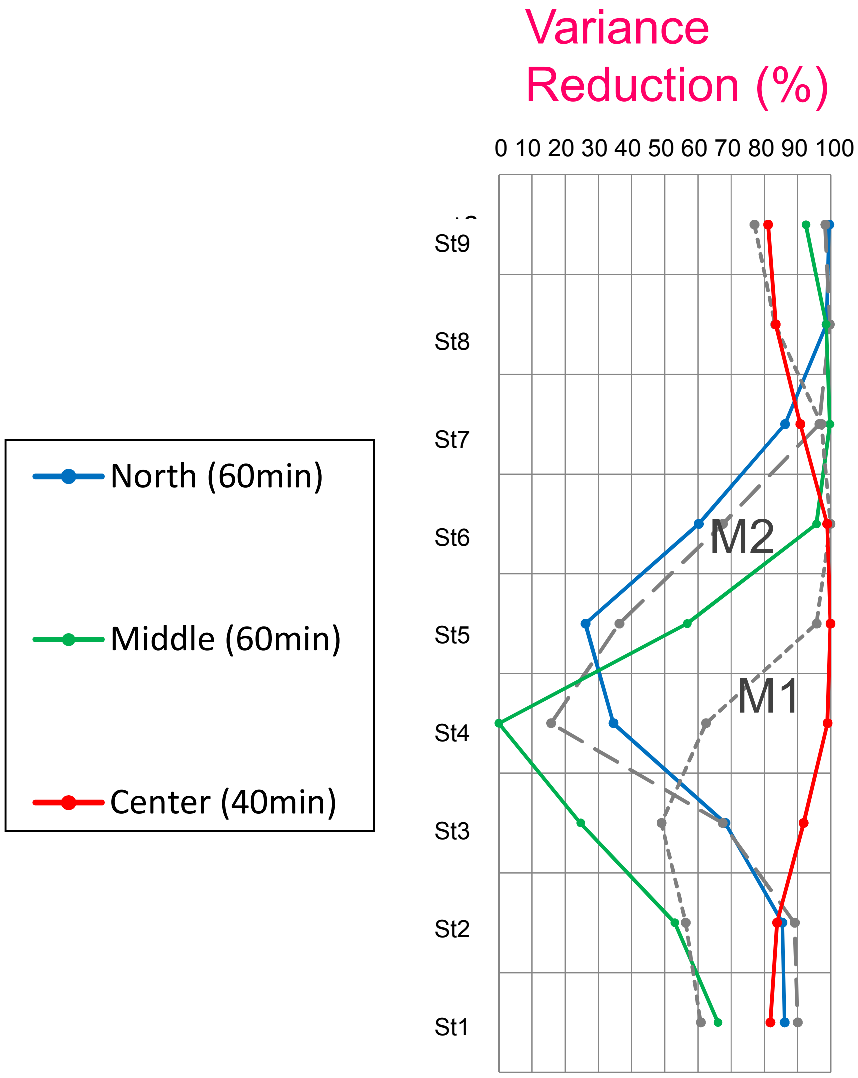


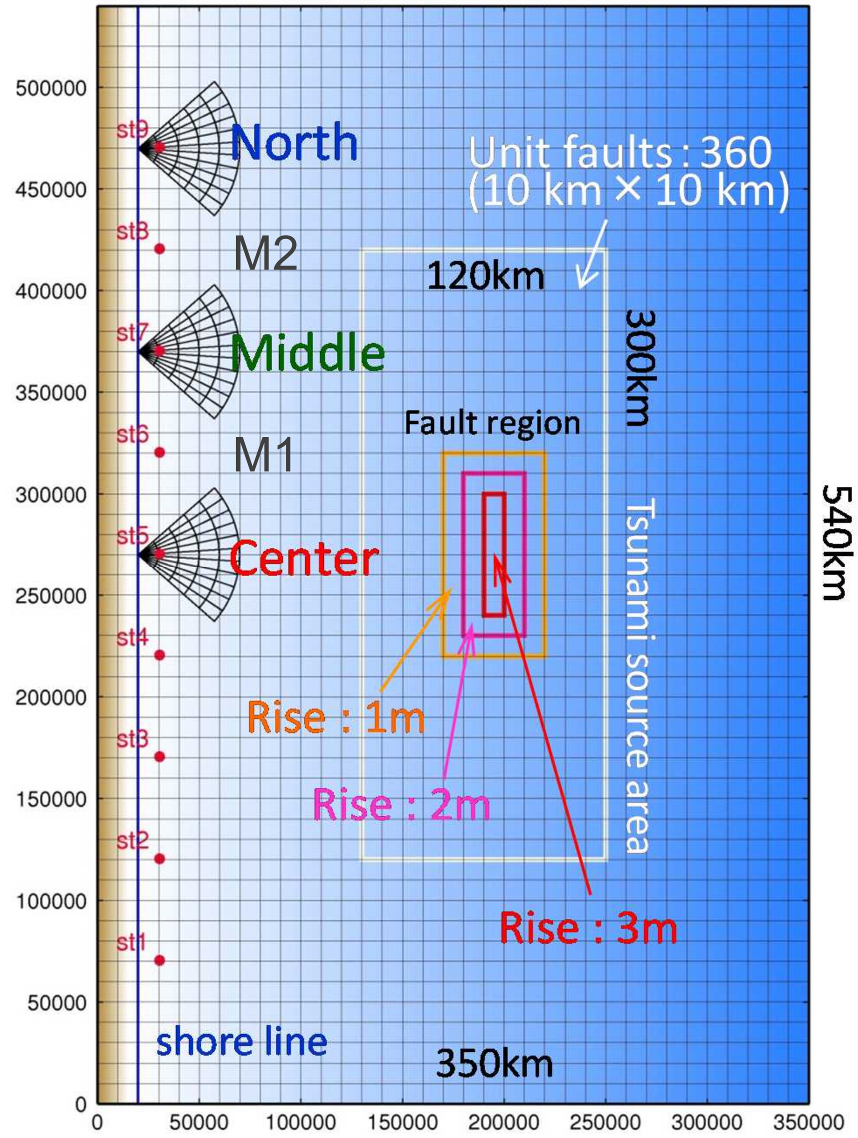


RMSE (m)	
0.178	
Variance Reduction (%) (6 hours)	
St9	92.5
St8	98.6
St7	99.7
St6	95.7
St5	56.7
St4	0.0
St3	24.6
St2	53.0
St1	66.0

Middle

Inversion Result ③ (Radar Location)





Inversion Result

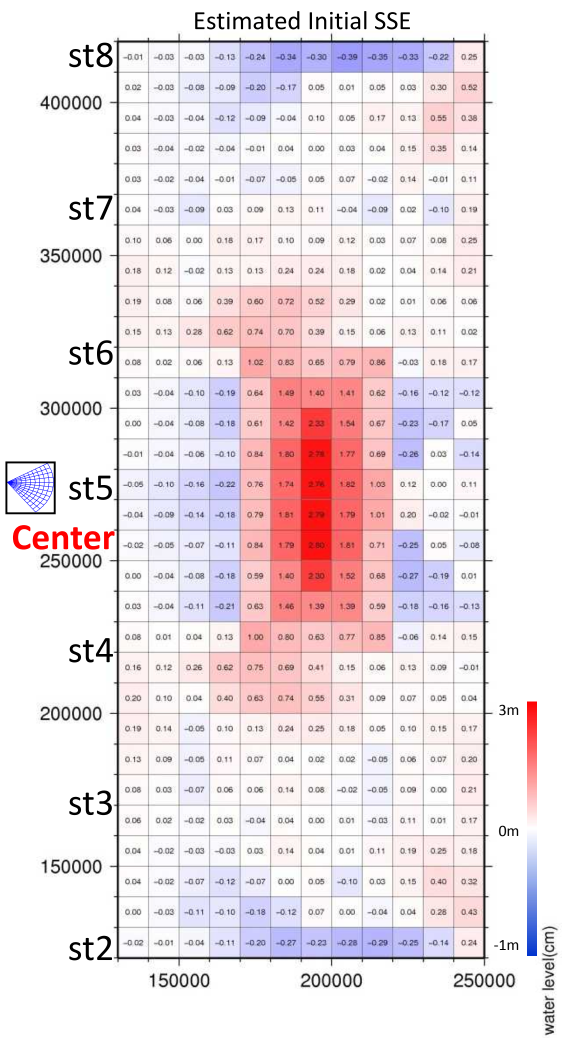
influences of

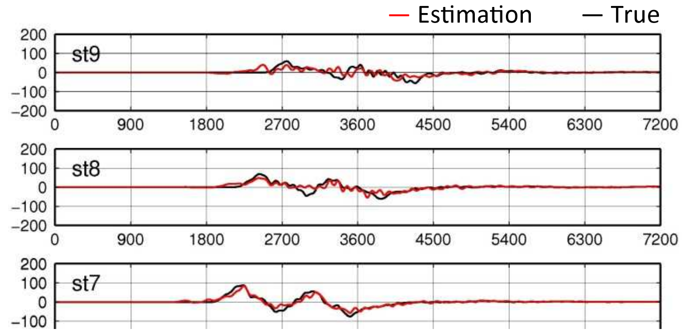
Spatial and Temporal Averaging

Inversion Result ④ (Spatial Averaging)

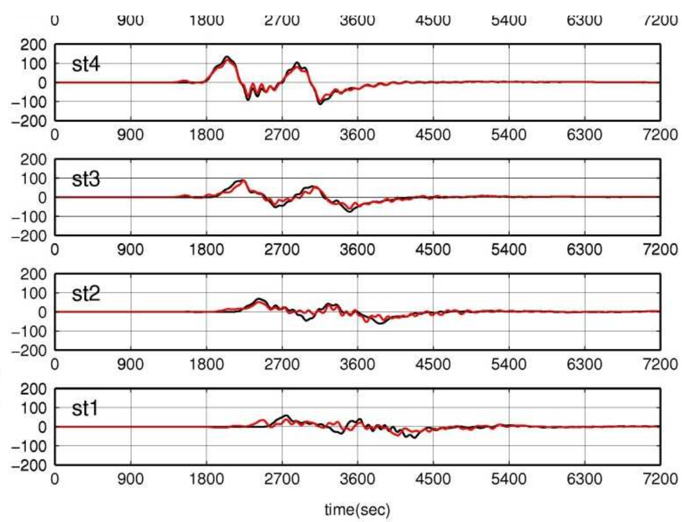
(1) Radar location : **Center**
(2) Spatial average : Yes
(3) Sampling interval : 10 sec

➢ Data sampling period : 40min



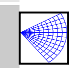


Still good compared to no spatial averaging results



RMSE (m)
0.099 (0.069)

Variance Reduction (%) (6 hours)	
St9	30.6 (81.1)
St8	67.9 (83.4)
St7	87.9 (90.8)
St6	95.4 (98.9)
St5	99.8 (99.9)
St4	96.0 (99.0)
St3	86.9 (91.8)
St2	69.2 (83.8)
St1	31.3 (81.8)



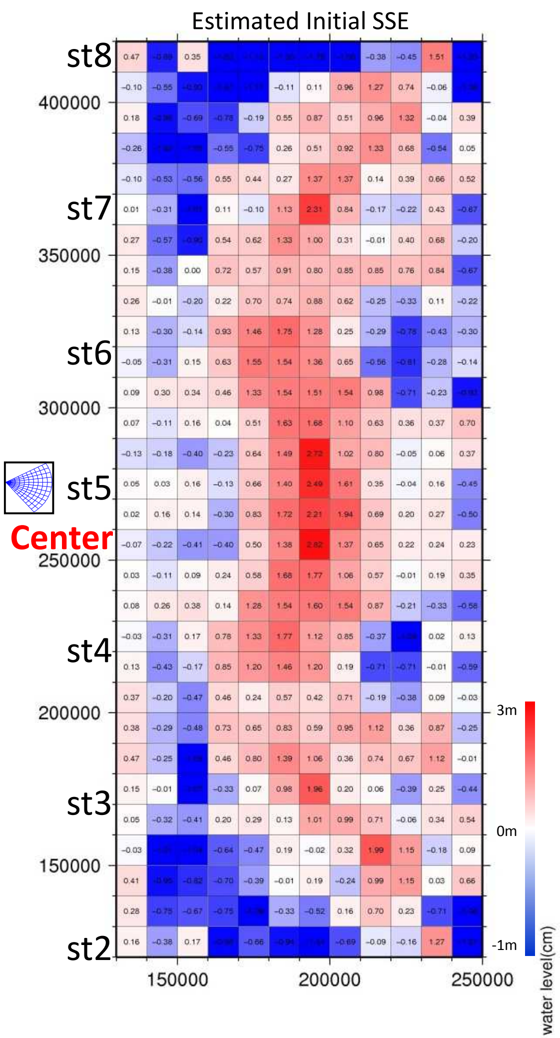
Center

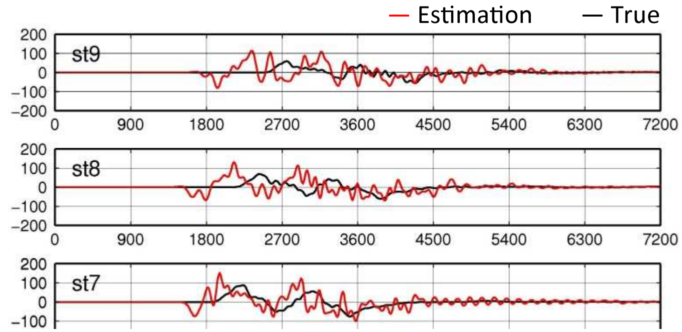
( ) : Center, No Spatial Average

Inversion Result ⑤ (Spatial and Temporal Averaging)

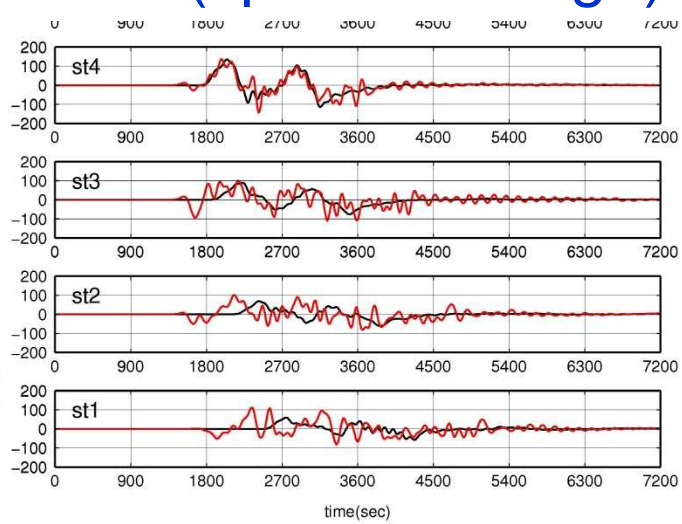
(1) Radar location : **Center**
(2) Spatial average : Yes
(3) 2min Temporal average, Sampling interval : 1 min

➢ Data sampling period : 40min





Become worse compared to previous result (spatial average)



RMSE (m)
0.294 (0.069)

Variance Reduction (%)	(6 hours)
St9	0.00 (30.6)
St8	0.00 (67.9)
St7	0.00 (87.9)
St6	73.8 (95.4)
St5	97.6 (99.8)
St4	68.5 (96.0)
St3	0.00 (86.9)
St2	0.00 (69.2)
St1	0.00 (31.3)

Center

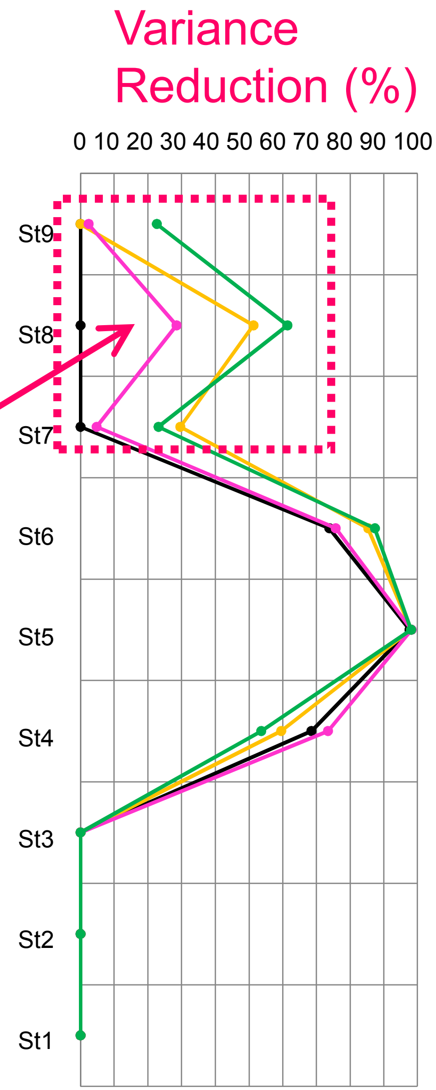
Inversion Result ⑨ (Spatial and Temporal Averaging)

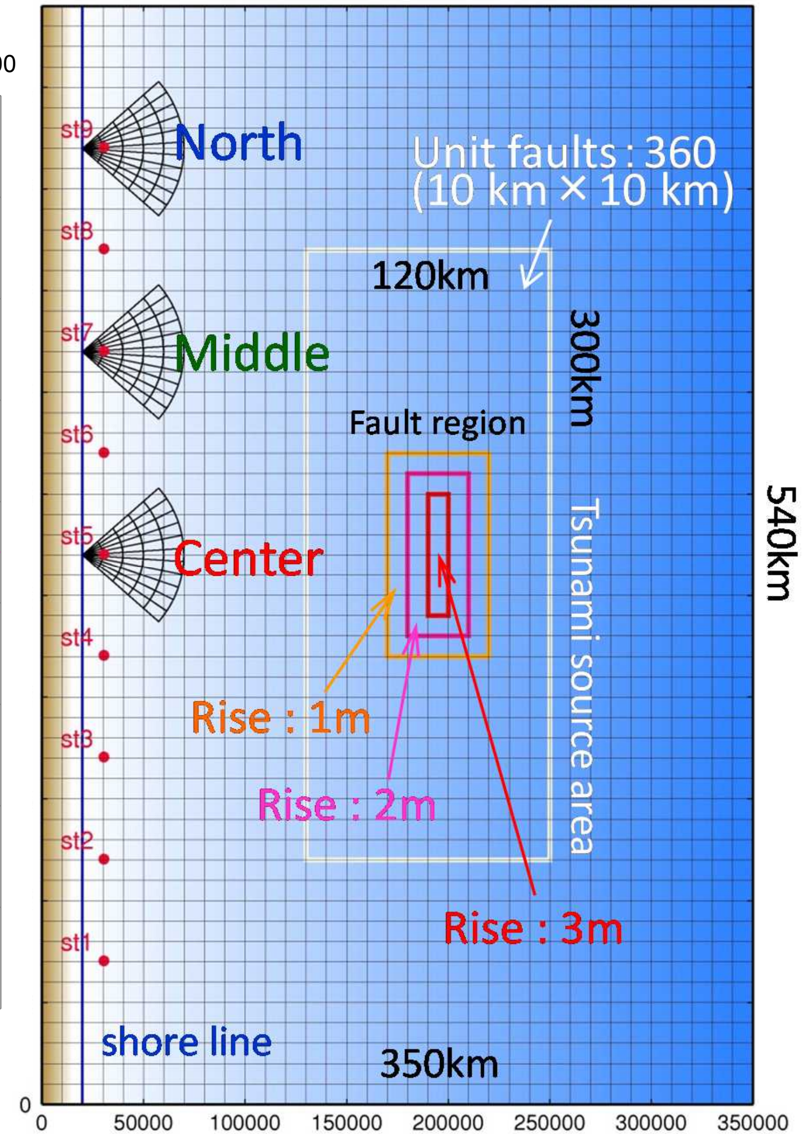
Influences of

➢ Radar Combination

significantly recovered around the Middle-North

Spatial Average
2min Temporal Average, Sampling interval : 1min (Center)





Conclusion and Future Study

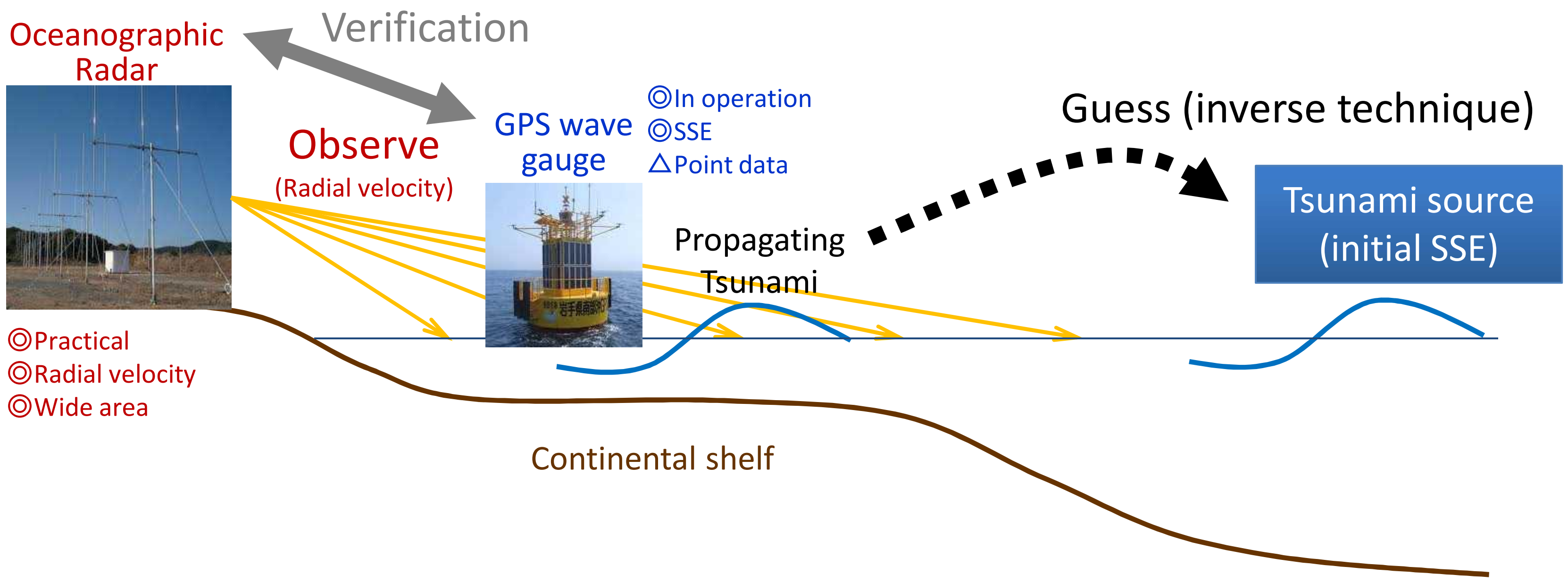
Through this study...

- Temporal averaging process largely affects inversion results compared to spatial averaging process
- Although Variance Reductions decrease due to temporal averaging process, maximum wave heights and their appearance time seem to agree well
- (As for Radar, it seems necessary to get high resolution data for Tsunami waveform inversion)

- *We would like to evaluate these influences quantitatively*
- *We are trying to reveal the mechanism of these influences in terms of Energy Flux and Wave Ray*
- *Also we are considering to estimate influences of red / white noise etc.*

Backup Slides

Application of Oceanographic Radar



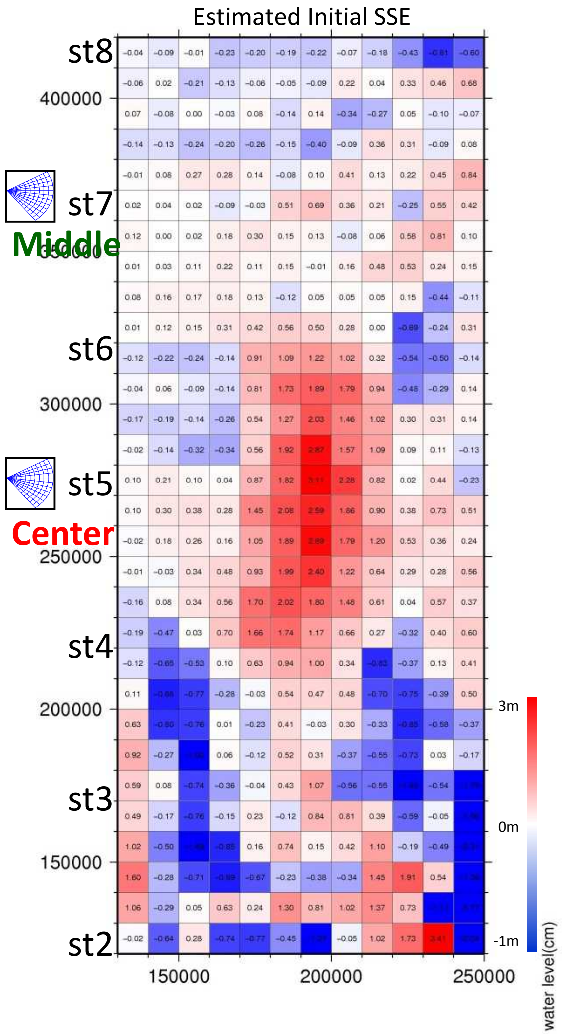
Inversion Result ⑥ (Spatial and Temporal Averaging)

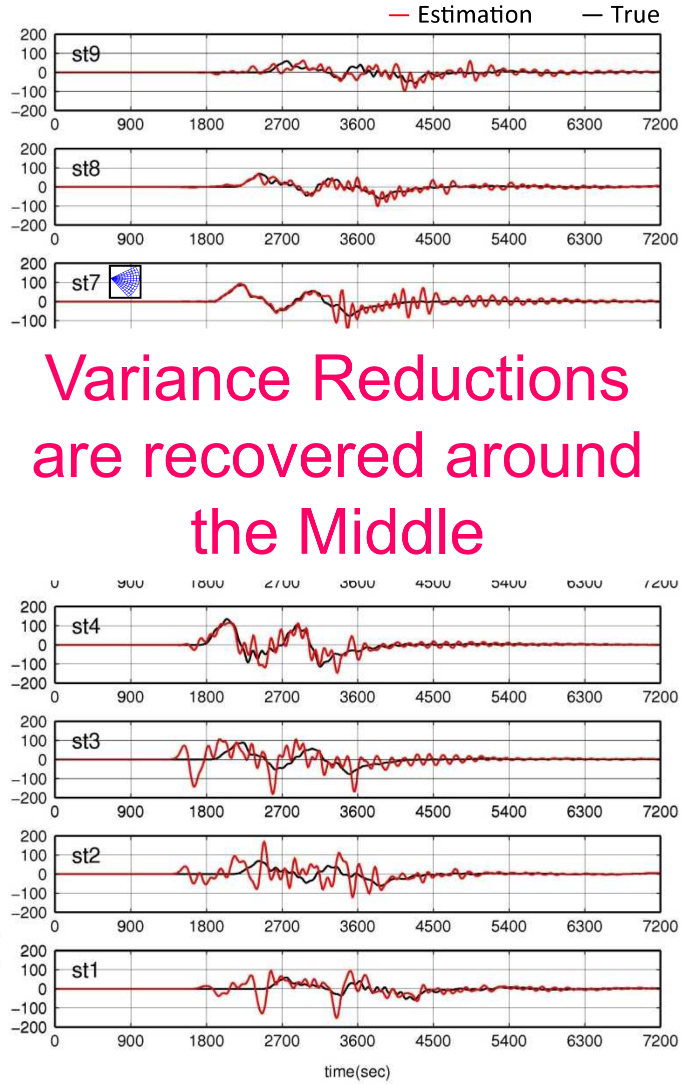
(1) Radar location : **Center** + **Middle**

(2) Spatial average : Yes

(3) 2min Temporal average, Sampling interval : 1 min

➤ Data sampling period : 40min





Variance Reductions are recovered around the Middle

RMSE (m)
0.255 (0.069)

Variance Reduction (%) (6 hours)

St9	0.00 (0.00)
St8	51.3 (0.00)
St7	29.6 (0.00)
St6	85.3 (73.8)
St5	98.1 (97.6)
St4	59.5 (68.5)
St3	0.00 (0.00)
St2	0.00 (0.00)
St1	0.00 (0.00)

Middle

Center

( ) : Center
Spatial & Temporal Average

Inversion Result ⑦ (Spatial and Temporal Averaging)

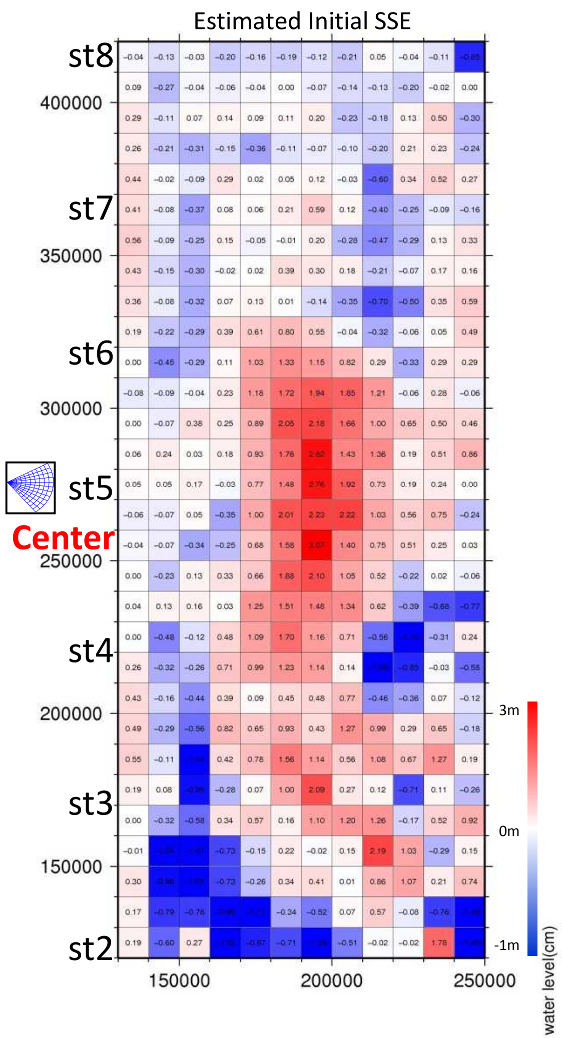
(1) Radar location : **Center** + **North**

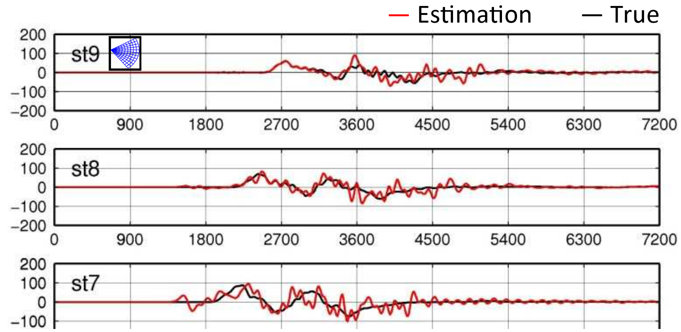
(2) Spatial average : Yes

(3) 2min Temporal average, Sampling interval : 1 min

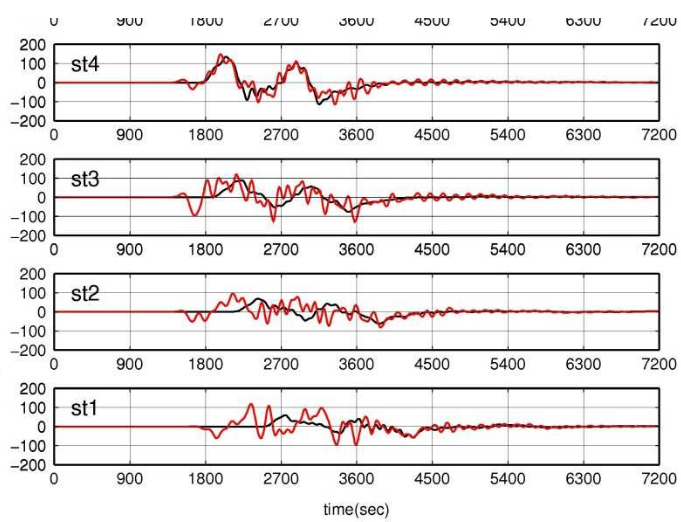
➢ Data sampling period : 40min

North





Variance Reductions are recovered around the North



RMSE (m)
0.228 (0.069)

Variance Reduction (%)	(6 hours)
St9	2.4 (0.00)
St8	28.5 (0.00)
St7	4.8 (0.00)
St6	75.7 (73.8)
St5	98.2 (97.6)
St4	73.4 (68.5)
St3	0.00 (0.00)
St2	0.00 (0.00)
St1	0.00 (0.00)

North

Center

( ) : Center
Spatial & Temporal Average

Inversion Result ⑧ (Spatial and Temporal Averaging)

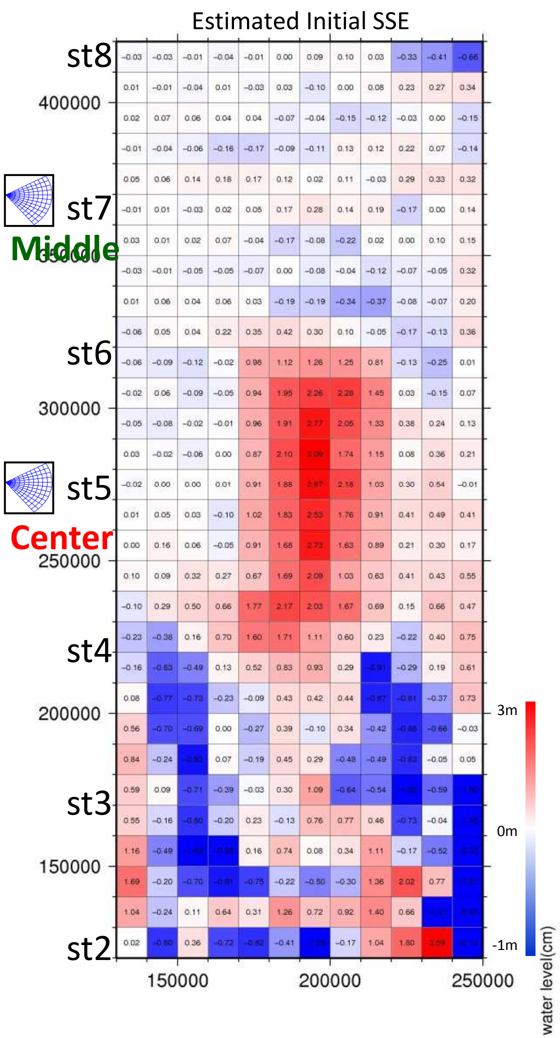
(1) Radar location : **Center** + **Middle** + **North**

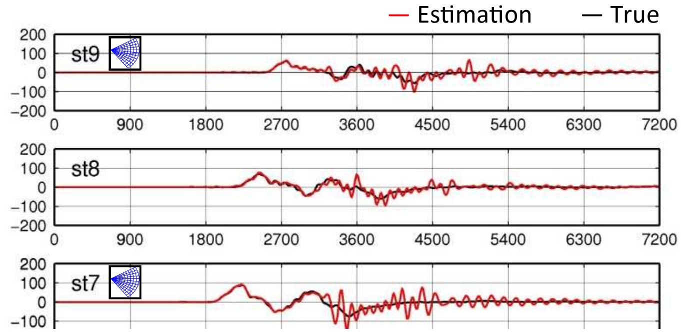
➢ Data sampling period : 40min

(2) Spatial average : Yes

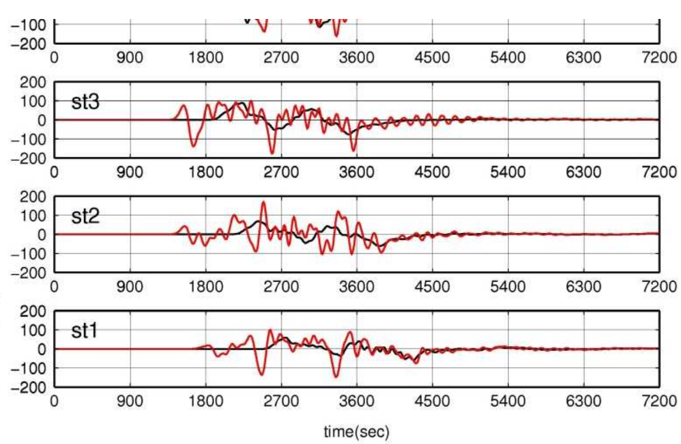
(3) 2min Temporal average, Sampling interval : 1 min

North





Variance Reductions are significantly recovered around the Middle - North



RMSE (m)
0.249 (0.069)

Variance Reduction (%)	(6 hours)
St9	22.6 (0.00)
St8	61.4 (0.00)
St7	23.1 (0.00)
St6	87.3 (73.8)
St5	98.1 (97.6)
St4	53.6 (68.5)
St3	0.00 (0.00)
St2	0.00 (0.00)
St1	0.00 (0.00)

North

Middle

Center

( ) : Center Spatial & Temporal Average

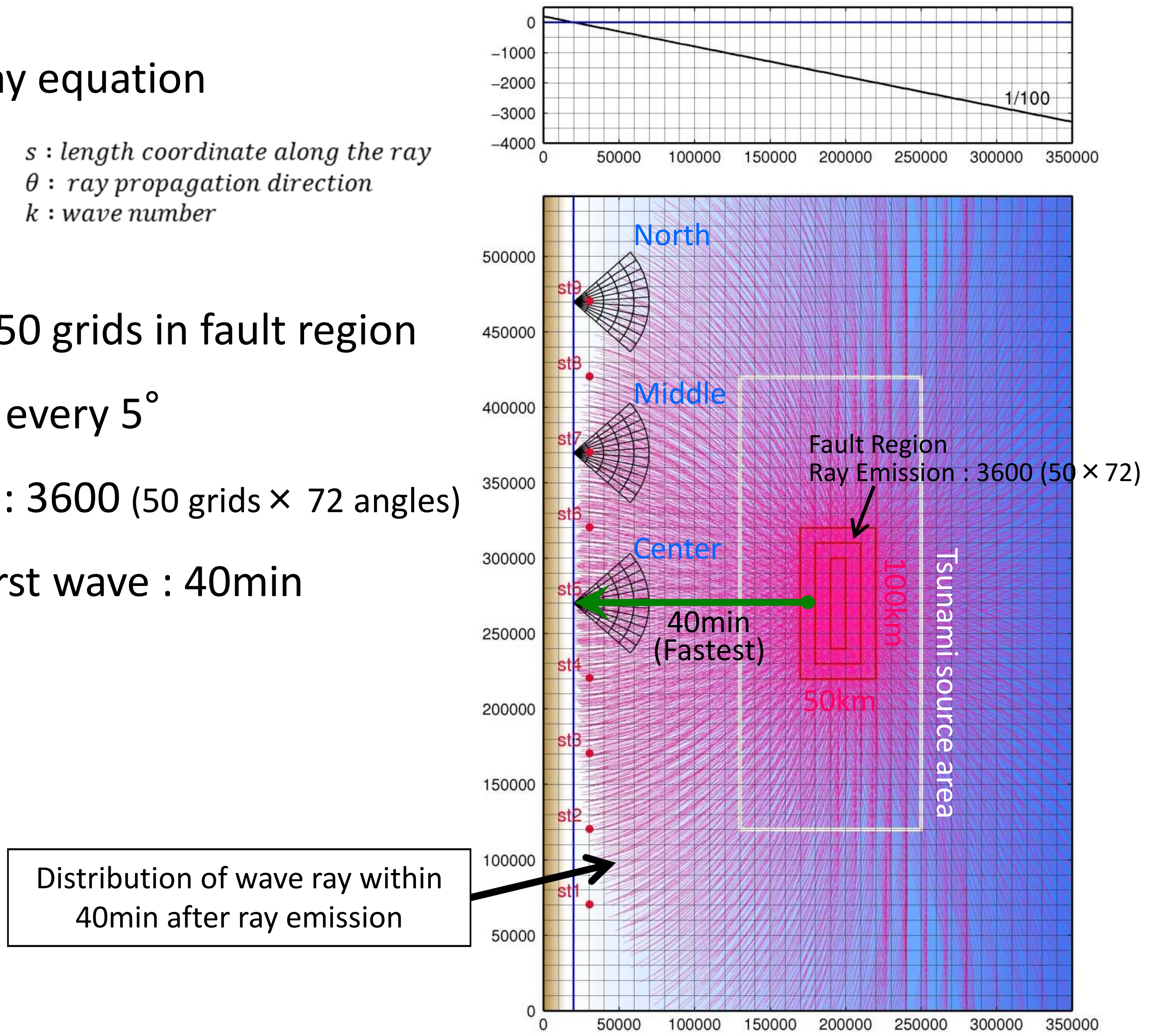
Wave Ray Calculation

- Method : applying ray equation

$$\frac{d\vartheta}{ds} = -\frac{1}{k}\left(sin\theta\frac{\partial k}{\partial x} - cos\theta\frac{\partial k}{\partial y}\right)$$

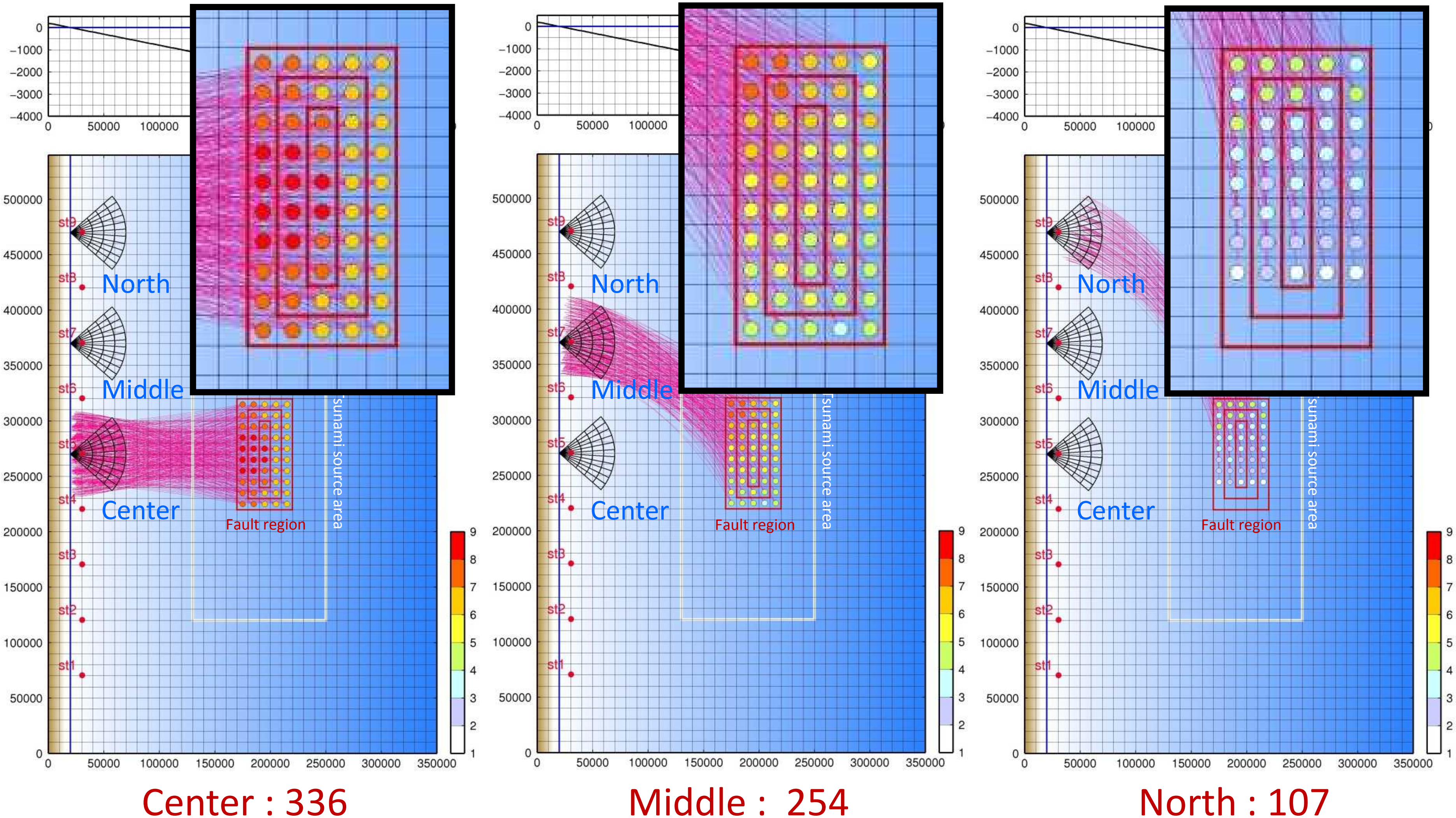
s : length coordinate along the ray
θ : ray propagation direction
k : wave number

- Ray emission : from 50 grids in fault region
- Ray emission angle : every 5˚
- Total number of Ray : 3600 (50 grids × 72 angles)
- Arrival time of the first wave : 40min



Wave Ray Calculation

- Number of ray reach Rader observation area
  (at a range of 50km from shoreline)



● : Emission source of ray reached Radar area within 40 min