# 紀伊水道における海洋レーダーを用いた津波データ同化への不均一誤差分布の適用

Tsunami Data Assimilation in The Kii Channel, Japan by Oceanographic Radar; Incorporation of Beam Angle-Dependent Measurement Errors

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# Application of High-Frequency Radar for Assimilating Tsunami Wavefield

*Data Assimilation*

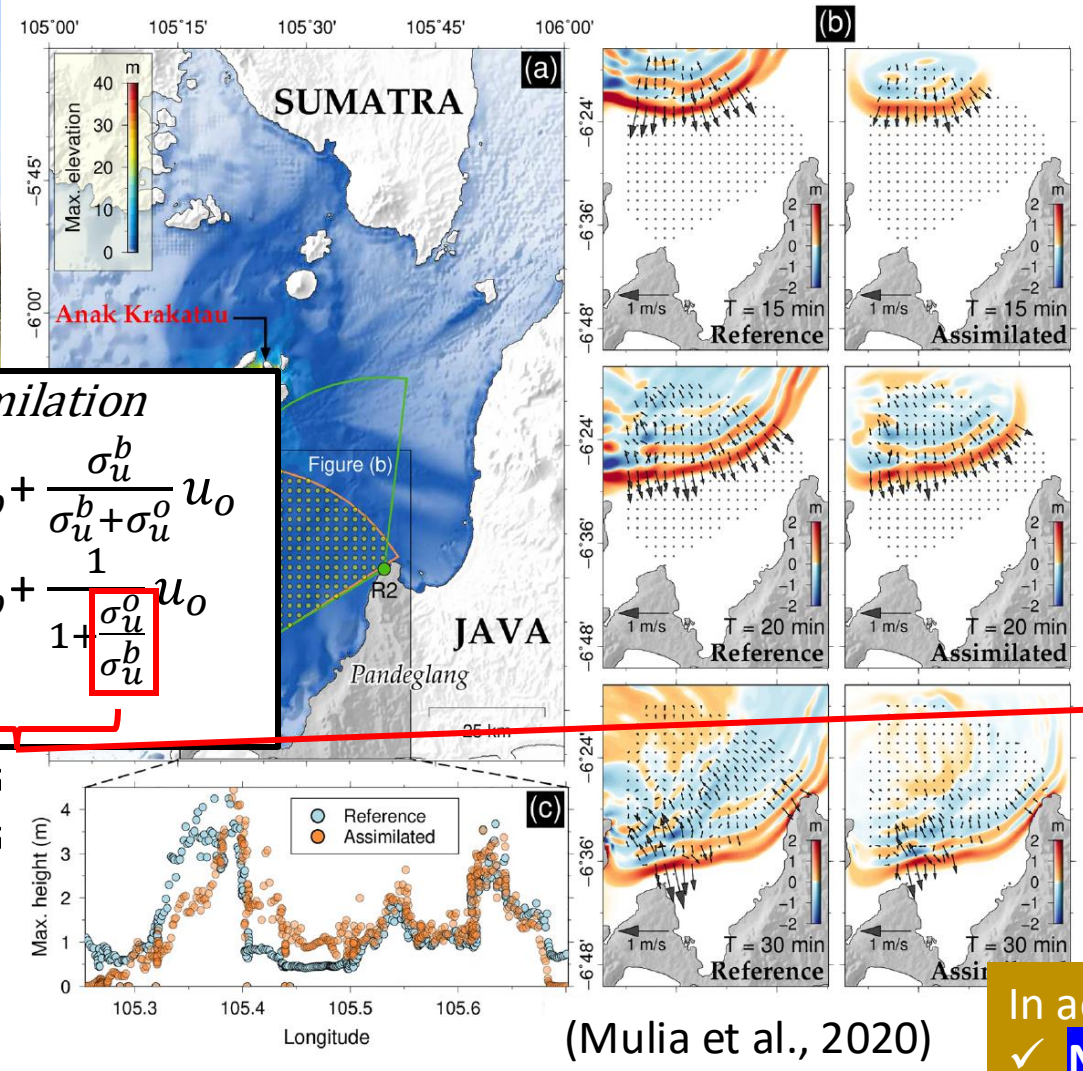
$$u_a = \frac{\sigma_u^o}{\sigma_u^b+\sigma_u^o} u_b + \frac{\sigma_u^b}{\sigma_u^b+\sigma_u^o} u_o$$

$$u_a = \frac{1}{1+\frac{\sigma_u^b}{\sigma_u^o}} u_b + \frac{1}{1+\frac{\sigma_u^o}{\sigma_u^b}} u_o$$

- Real-time tsuna
  (Fuji and Hinata
  2018)

**Tsunami forecasti**

- Assimilating ra
  (Mulia et al., 2
  2023)




(Mulia et al., 2020)

**Figure 4.** (a) Maximum simulated tsunami elevation and hypothetical radar observations coverage. R1 and R2 the radar locations. (b) Simulated tsunami wavefield used as the reference (left panel) and assimilation result panel). (c) Comparisons between the reference and the assimilated maximum coastal tsunami heights.

**Successfully predicting tsunami wavefield using optimal interpolation method for radar-derived surface currents.**

- ✓ Synthesizing **20% of Gaussian noise** to the simulated EW and NS velocities.
- ✓ Achieving **an 85 % forecast accuracy** of the maximum coastal tsunami height.

Within OI method, **assimilation performance depends on the parameter $\rho = \sigma^o/\sigma^b$** (Maeda et al., 2015)

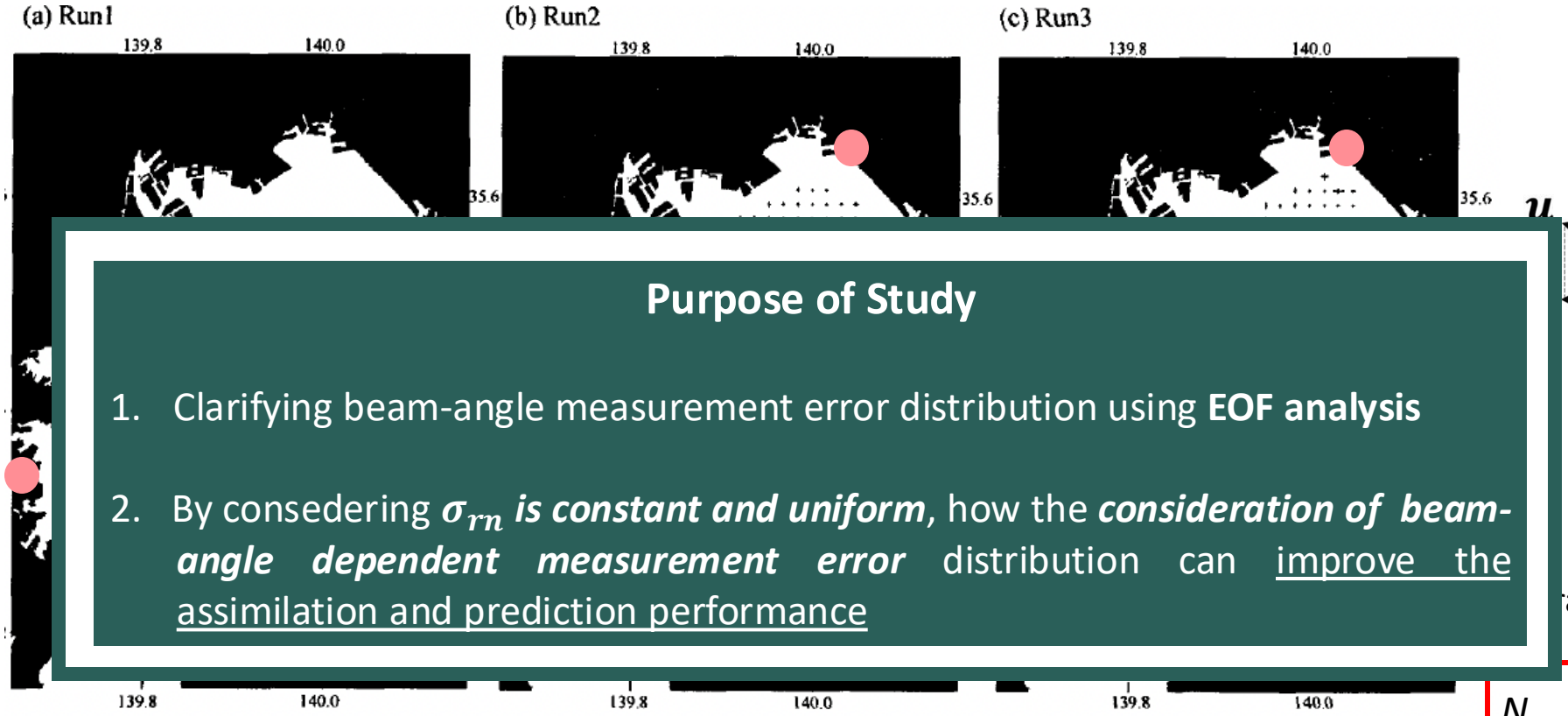
**Issue on Previous Studies:**

Applying **time-independent and uniform measurement error distribution ($\sigma_u^o, \sigma_v^o$)** within the OI method which is $\rho = \sigma^o/\sigma^b = 1$

In actuality, measurement errors ($\sigma_u^o$, $\sigma_v^o$) by HF radar are:

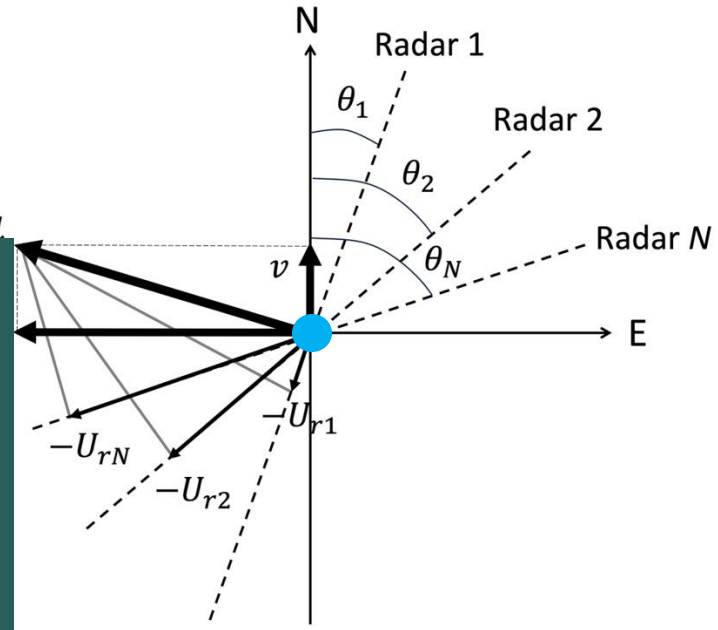
- ✓ **Not spatiotemporaly uniform**
  (Nadai et al., 1999; Hinata et al., 2005)

# Factors Influencing Current Measurement by HF Radar




## Purpose of Study

1. Clarifying beam-angle measurement error distribution using **EOF analysis**
2. By consedering ***$\sigma_{rn}$ is constant and uniform***, how the ***consideration of beam-angle dependent measurement error*** distribution can improve the assimilation and prediction performance




adar geometry in a measurement point

(Hinata et al., 2005)

$$\begin{pmatrix}\sigma_u^2\\ \sigma_v^2\end{pmatrix} = \frac{1}{\sum_{n=1}^{N}\frac{\sin^2\theta_n}{\sigma_{rn}^2}\times\sum_{n=1}^{N}\frac{\cos^2\theta_n}{\sigma_{rn}^2}-\left[\sum_{n=1}^{N}\frac{\sin\theta_n\cos\theta_n}{\sigma_{rn}^2}\right]^2}\times\begin{bmatrix}\sum_{n=1}^{N}\frac{\cos^2\theta_n}{\sigma_{rn}^2}\\ \sum_{n=1}^{N}\frac{\sin^2\theta_n}{\sigma_{rn}^2}\end{bmatrix}$$

$N$ = **Number of radar**.

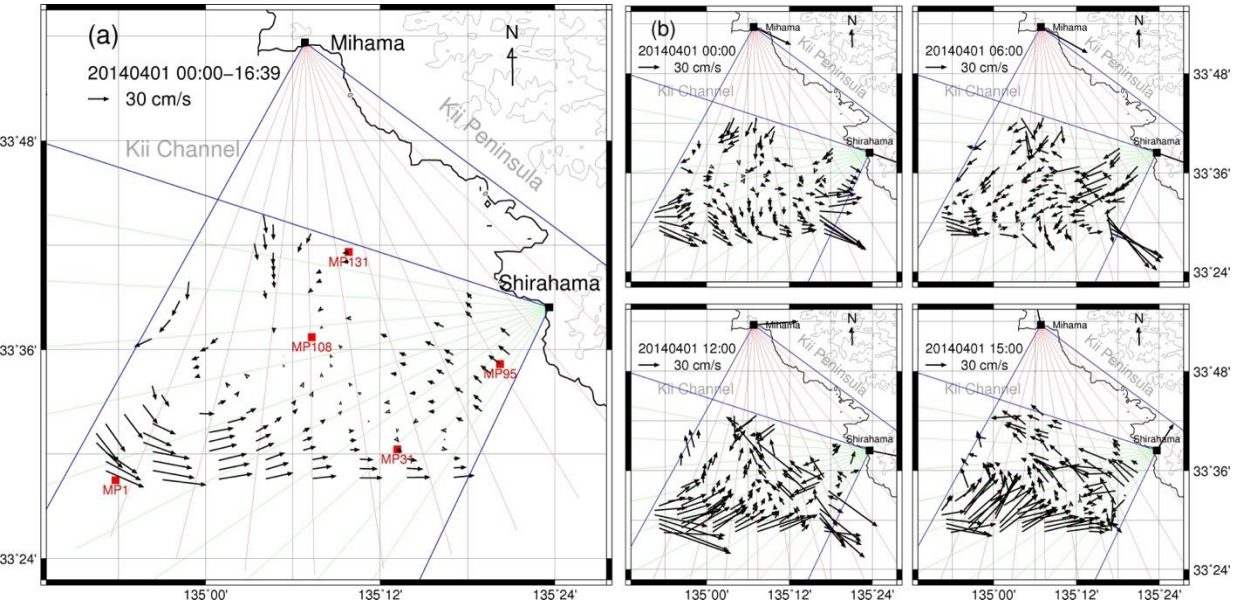
$\sigma_u, \sigma_v$ = East-west and north-south measurement errors.

$\sigma_{rn}$ = **Radial velocity error** of $n^{\text{th}}$ radar.

$\theta_n$ = **Direction of beam** to north of $n^{\text{th}}$ radar.

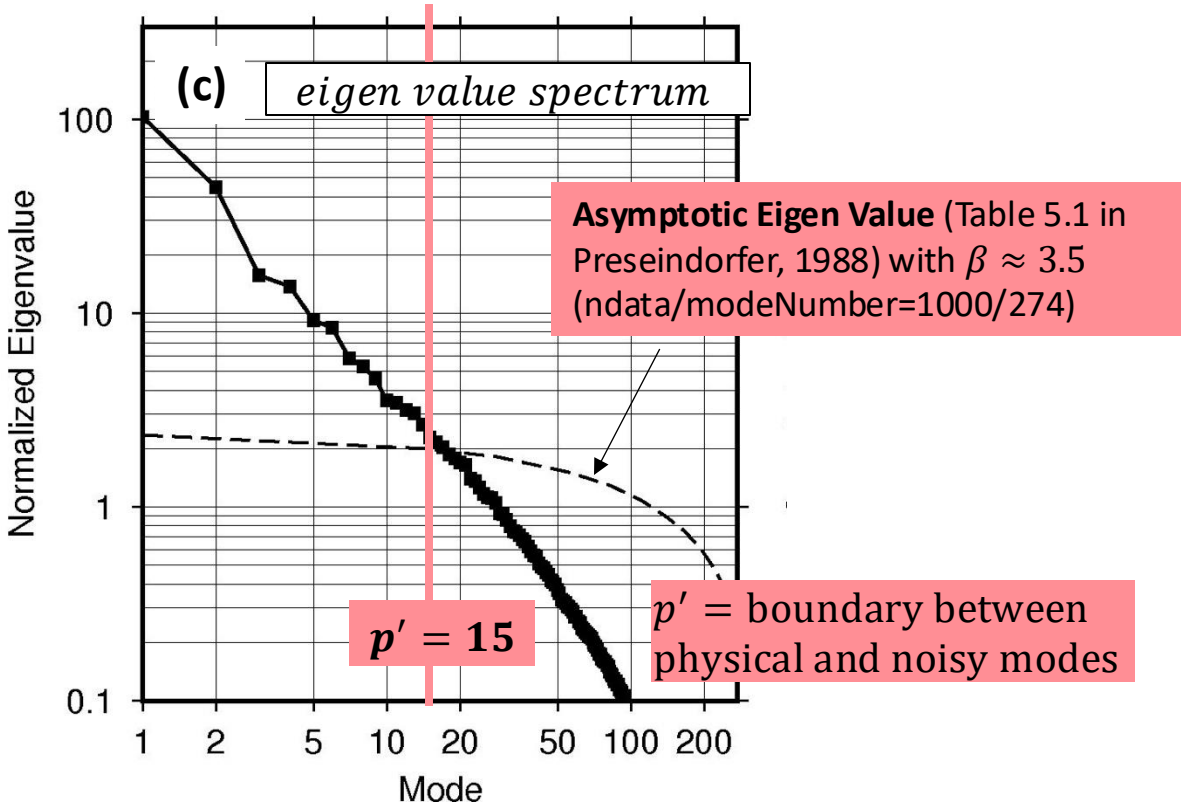
# ① EOF Analysis with Selection Rule N

Data Period → April 1st, 2014; 00:00-16:39 JST (1-min Interval Time) in Kii Channel (ndata=1000)




EOF Analysis with Rule N Selection Method

$$z(t,\boldsymbol{x}) == \underbrace{\sum_{j=1}^{p'} \ell_j^{\frac{1}{2}} \alpha_j(t) e_j(\boldsymbol{x})}_{\text{Physical Modes}} + \underbrace{\sum_{j=p'+1}^{p} \ell_j^{\frac{1}{2}} \alpha_j(t) e_j(\boldsymbol{x})}_{\text{Nosiy Modes}},$$




**Asymptotic Eigen Value** (Table 5.1 in Preseindorfer, 1988) with $\beta \approx 3.5$ (ndata/modeNumber=1000/274)

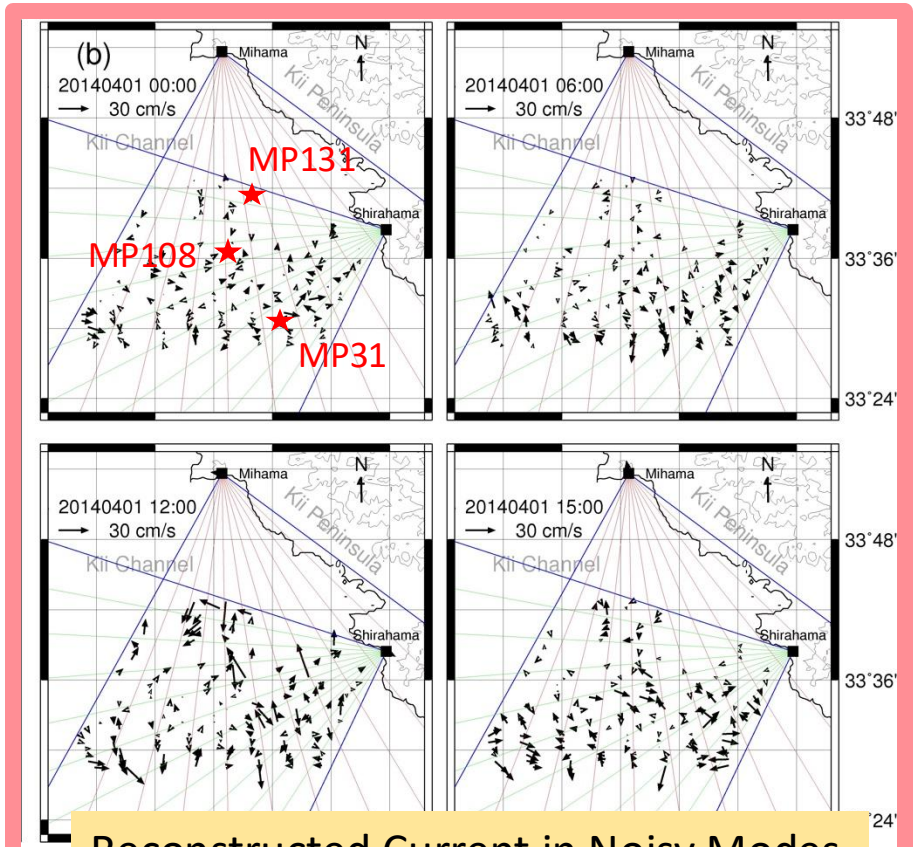
$p'$ = boundary between physical and noisy modes

$z(t,x)$ = Reconstructed EW and NS velocitiy components at measurement points ($p = 137 \times 2$)
$a_j(t)$ = Time dependent ampltude of mode $j$
$e_j(x)$ = The $j^{\text{th}}$ eigenfunction
$\ell_j$ = eigen value of mode $j$

(Sahana et al., 2024)

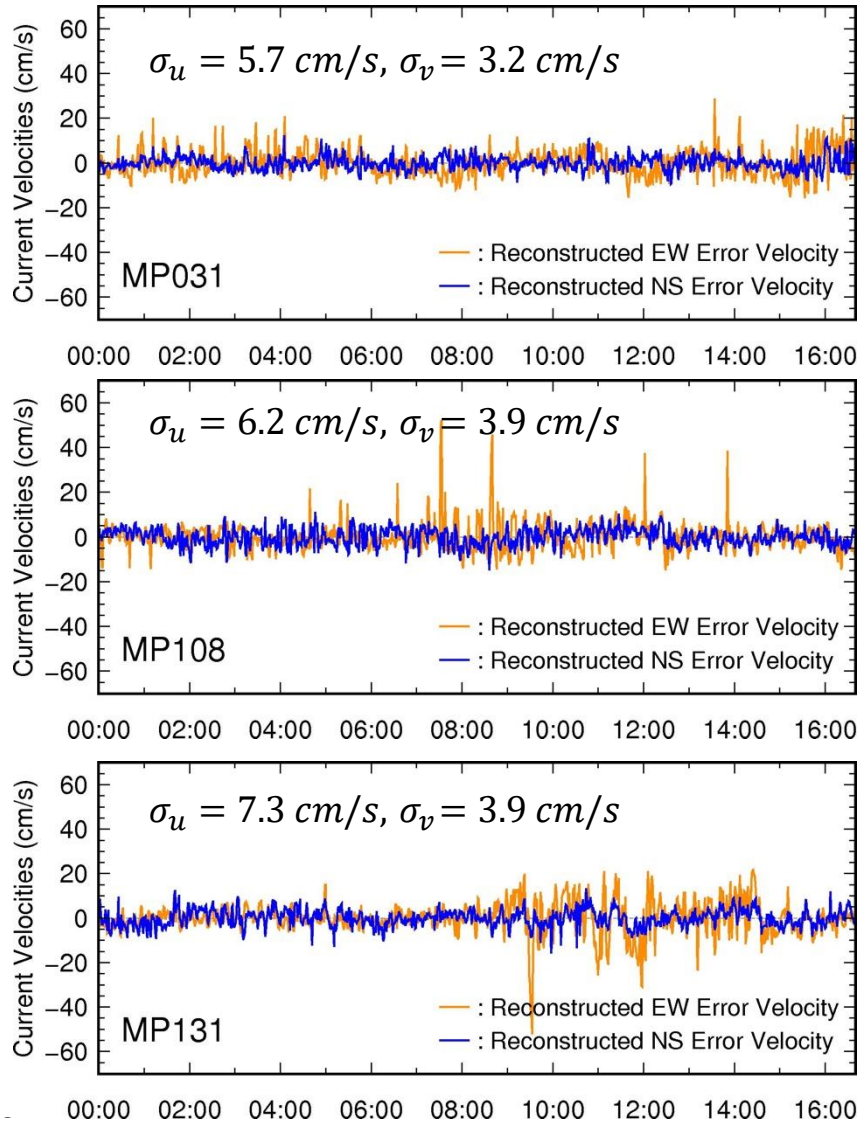
# ① Reconstructed Surface Current with Physical and Noise Field

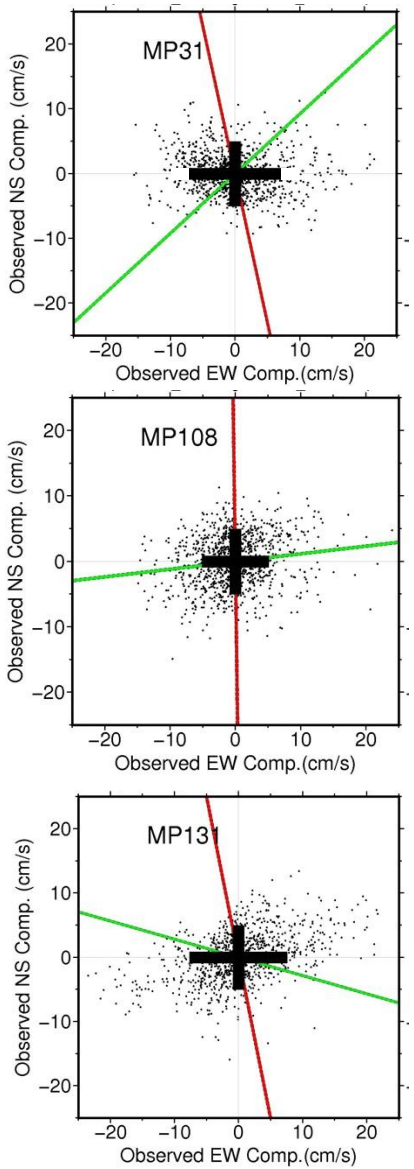
Data Period → April 1st, 2014; 00:00-16:39 JST (1-min Interval Time)




**Reconstructed Current in Noisy Modes**

- The current magnitude of noisy fields are **relatively much smaller** compared to physical fields
- **Inconsistent** motion




$\sigma_u \approx 2\,\sigma_v$

$\sigma_u \approx 2\,\sigma_v$

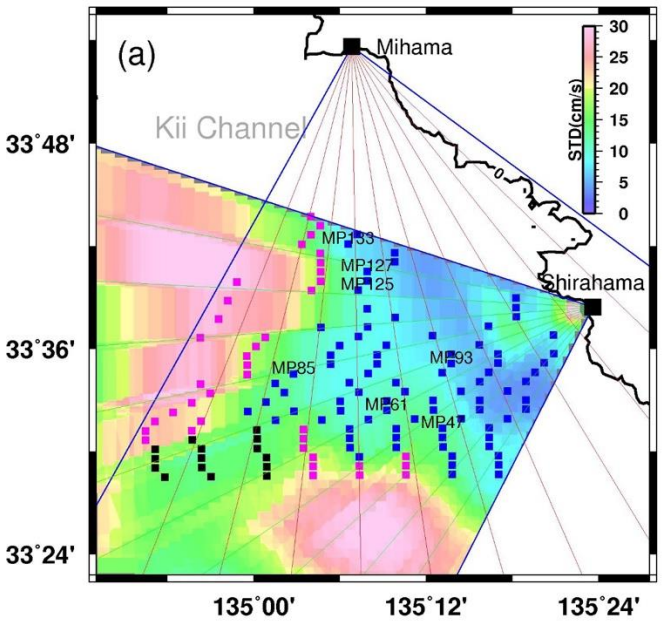
$\sigma_u \approx 2\,\sigma_v$

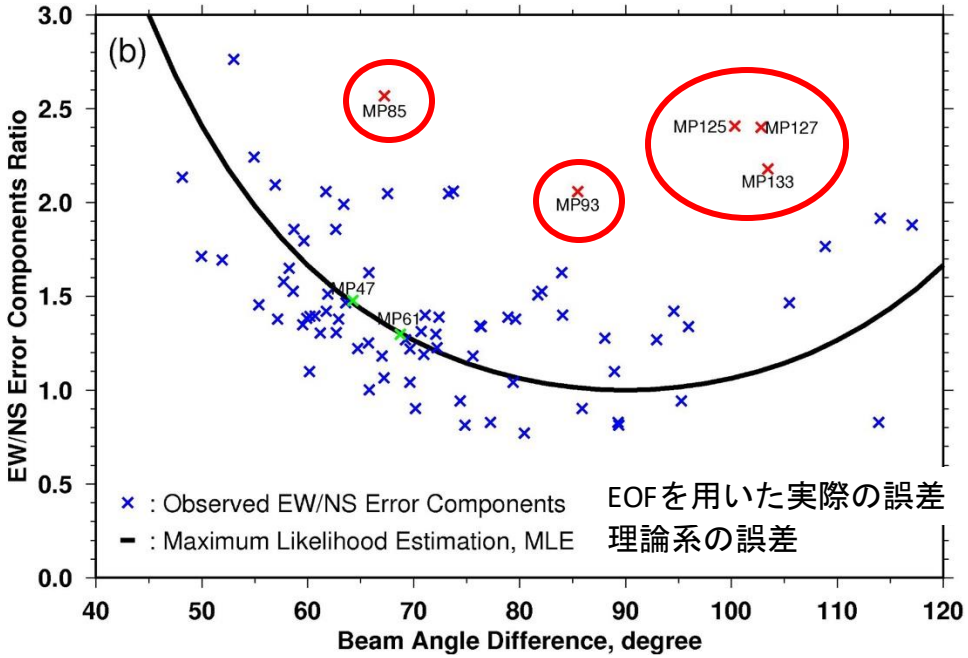
**Beam-Angle Dependent Measurement Error Distribution**

# ① Measurement Error by Observation and Theory

- Not all the 137 points show $\sigma_u/\sigma_v$ ratios that aligned with the theory.
- 53 points (**black** and **purple**) were excluded from the comparison.
  - ✓ The points where the SNR was less than 25 dB (**black** points)
  - ✓ The standard deviation of the radial velocity exceeded 15 cm/s (**purple** points), based on the Shirahama radar measurements.

理論系とEOFを用いた実際の観測誤差の比較



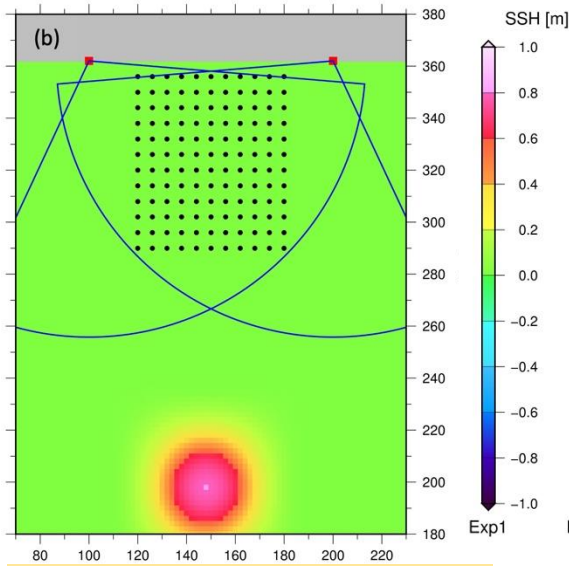



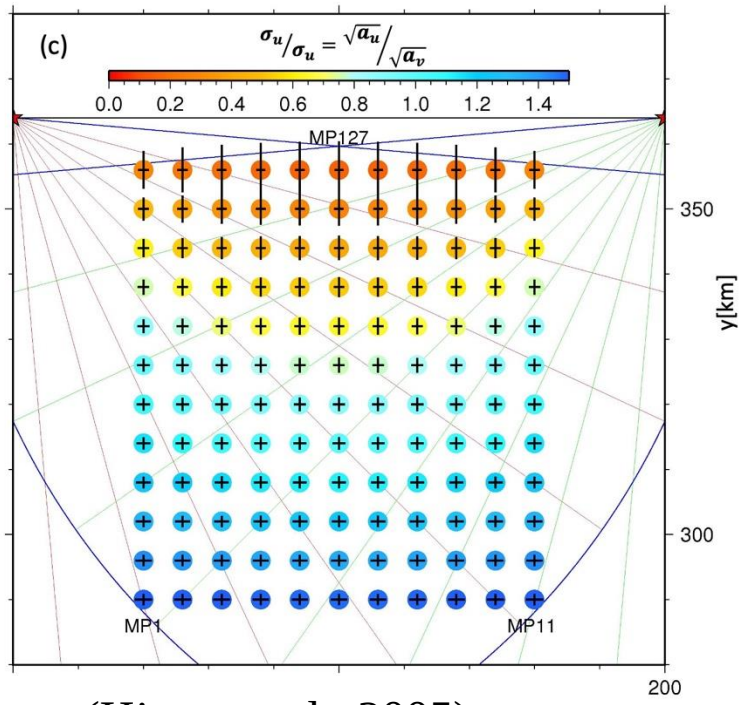

Since **the error measurements ($\sigma_u, \sigma_v$) is location dependent**, estimation of spatial distribution of $\sigma_u$ and $\sigma_v$ is **important for data assimilation**.

- 84 points (**blue**) were used for the comparison
  - ✓ Some points (MP85, MP93, MP133, MP125, MP127) **deviated** from the the
  - ✓ Most of points **aligned well** with the MLE theory

(Sahana et al., 2024)

# ② Estimation of Measurement Error ($\sigma_u, \sigma_v$) and U-V Assimilation by HF Radar




**1-m** initial tsunami height
**500 m** flat bottom

Estimation of Measurement Error (Hinata et al., 2005)

$$\begin{pmatrix}\sigma_{u,i}^o\\ \sigma_{v,i}^o\end{pmatrix}^2 = \frac{1}{\left[\sum_{n=1}^{N} cos^2\theta_{n,i} \times \sum_{n=1}^{N} sin^2\theta_{n,i}\right] - \left[\sum_{n=1}^{N} \sin\theta_{n,i}\cos\theta_{n,i}\right]^2}$$

$$\times \begin{pmatrix}\sum_{n=1}^{N} cos^2\theta_{n,i}\\ \sum_{n=1}^{N} sin^2\theta_{n,i}\end{pmatrix} \times \sigma_r^2 \qquad for\ \mathrm{N} = 2.$$

$\left(\sigma_{u,i}^o\right)^2 = a_{u,i} \times (\sigma_r^o)^2, \qquad \left(\sigma_{v,i}^o\right)^2$

$a$ =amplification factor

***Data Assimilation with Optimal Interpolation***

$$X_m^a = X_m^f + W\left[y_m - HX_m^f\right]$$

***Weight Calculation*** *with Consideration of Spatial Distribution of Measurement Errors (*$W_{gj}$ for U ≠ $W_{gj}$ for V*)*

$$\begin{cases}\sum_{j=1}^{132} w_{u,gj}\left(\mu_{ij}^b + \mu_{ij}^o \rho_{u,i}\rho_{u,j}\right) = \mu_{gi}^b\\ \sum_{j=1}^{132} w_{v,gj}\left(\mu_{ij}^b + \mu_{ij}^o \rho_{v,i}\rho_{v,j}\right) = \mu_{gi}^b\end{cases}$$

Nonuniformity of Obs Errors

Where

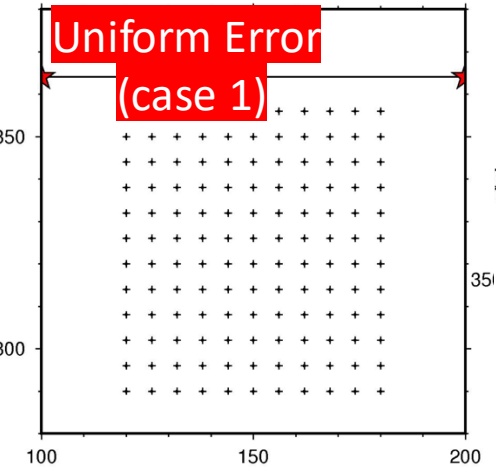
$$\rho_{u,i} = \frac{\sigma_{u,i}^o}{\sigma_{u,i}^b} = \frac{\sqrt{a_{u,i}} \times \sigma_r^o}{\sigma_r^o} = \sqrt{a_{u,i}}$$

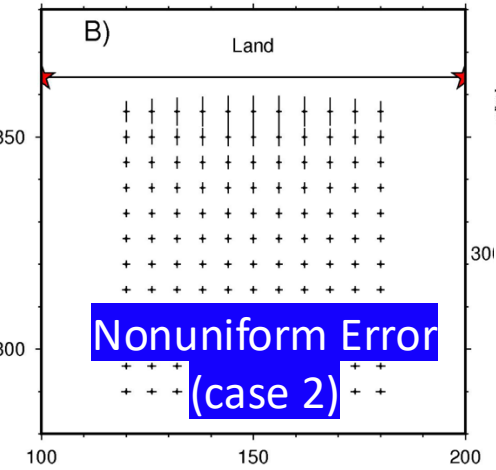
$$\rho_{v,i} = \frac{\sigma_{v,i}^o}{\sigma_{v,i}^b} = \frac{\sqrt{a_{v,i}} \times \sigma_r^o}{\sigma_r^o} = \sqrt{a_{v,i}}$$

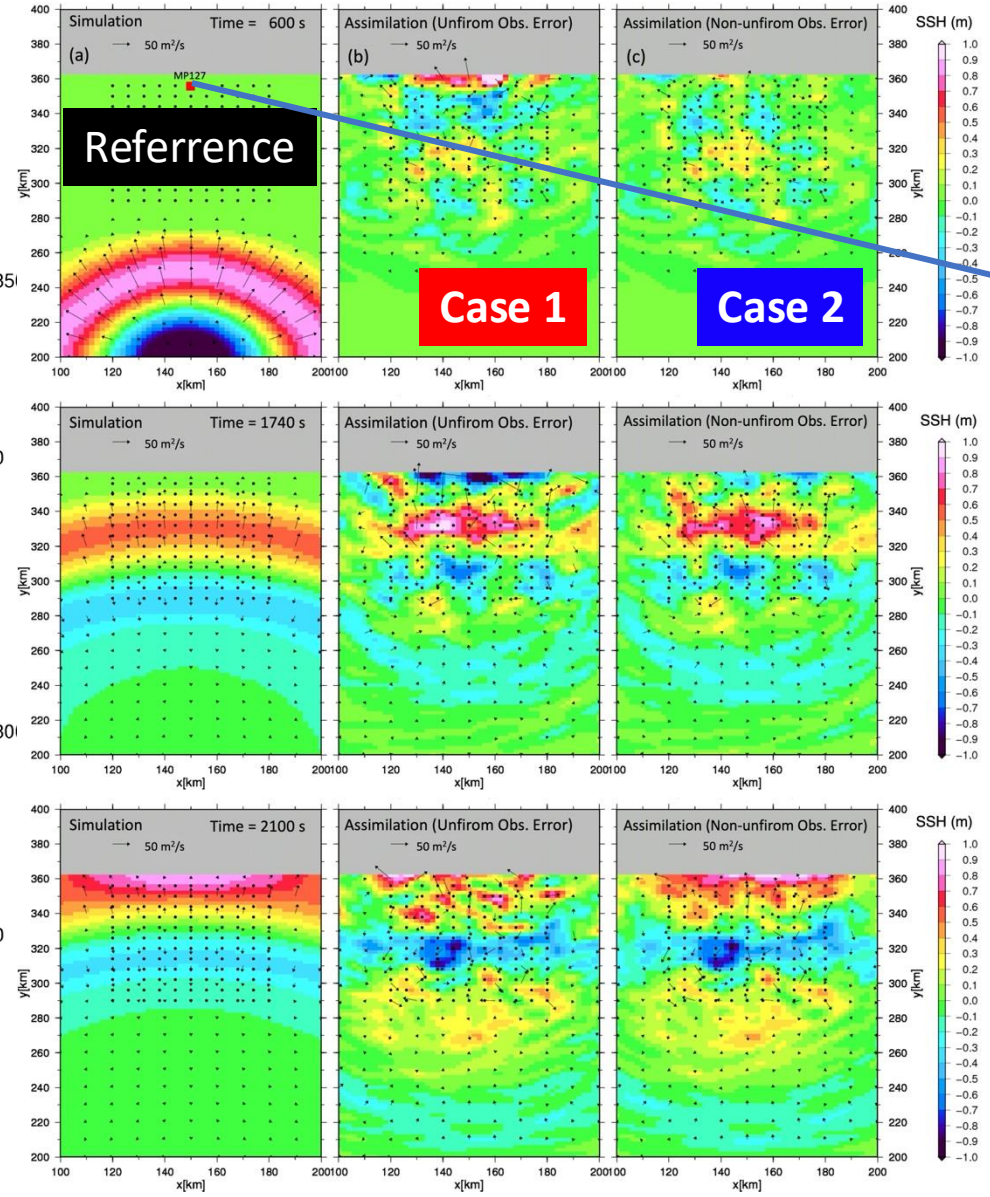
(Sahana et al., *2024*)

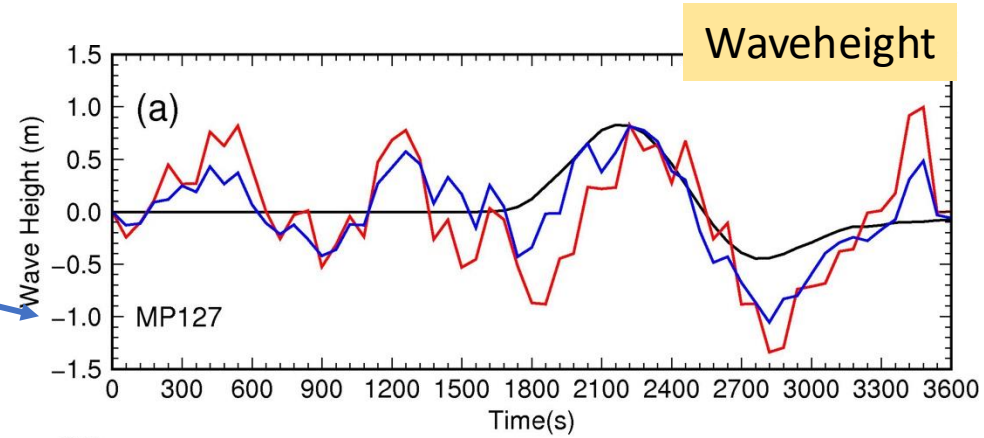
Incorporation of Measurement Error Distribution into Data Assimilation

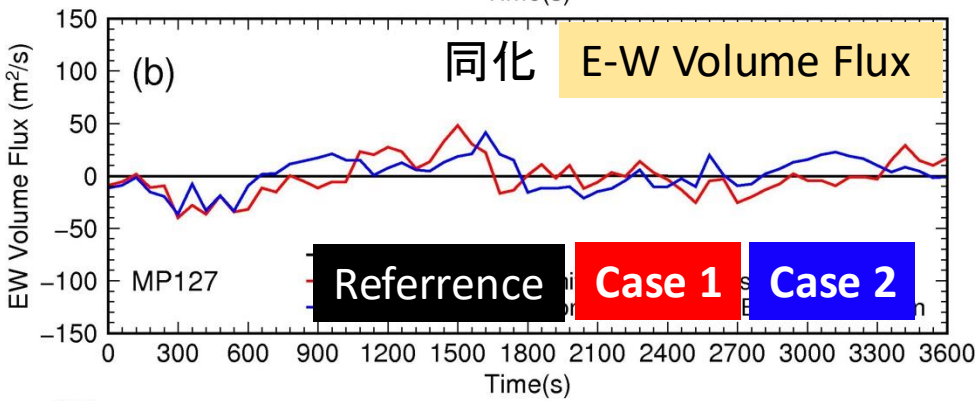
# ② Assimilation with Uniform and Non-Uniform Measurement Error (Exp. 1-1)

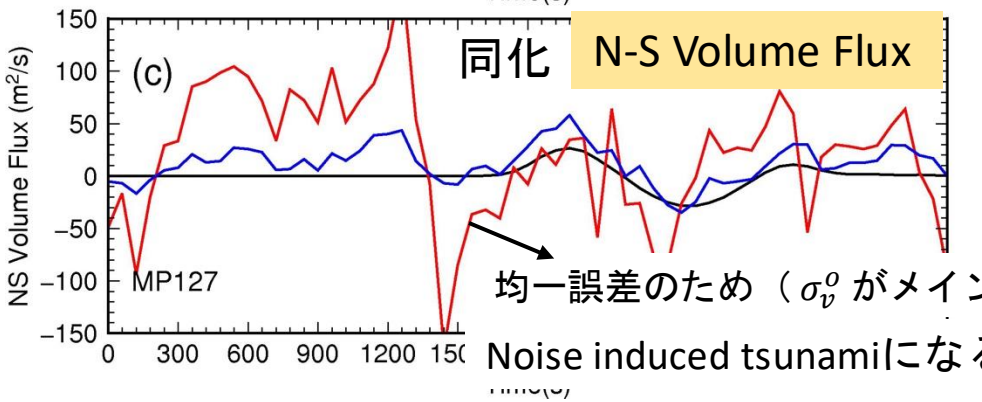


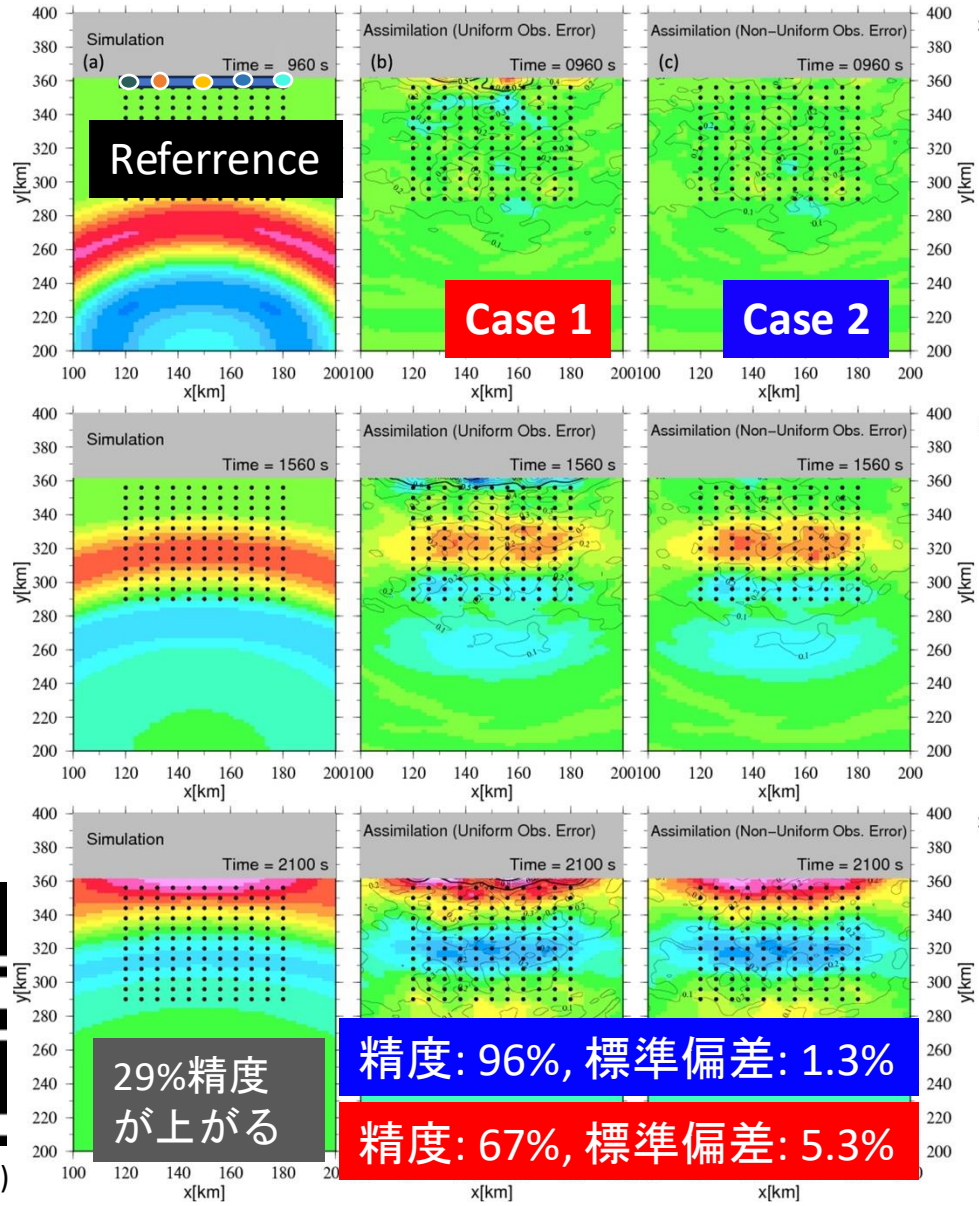




均一誤差のため（$\sigma_v^o$ がメイン）

Noise induced tsunamiになる

Incorporation of Measurement Error Distribution into Data Assimilation

# ② Assimilation with Uniform and Non-Uniform Measurement Error (15 Exps.)



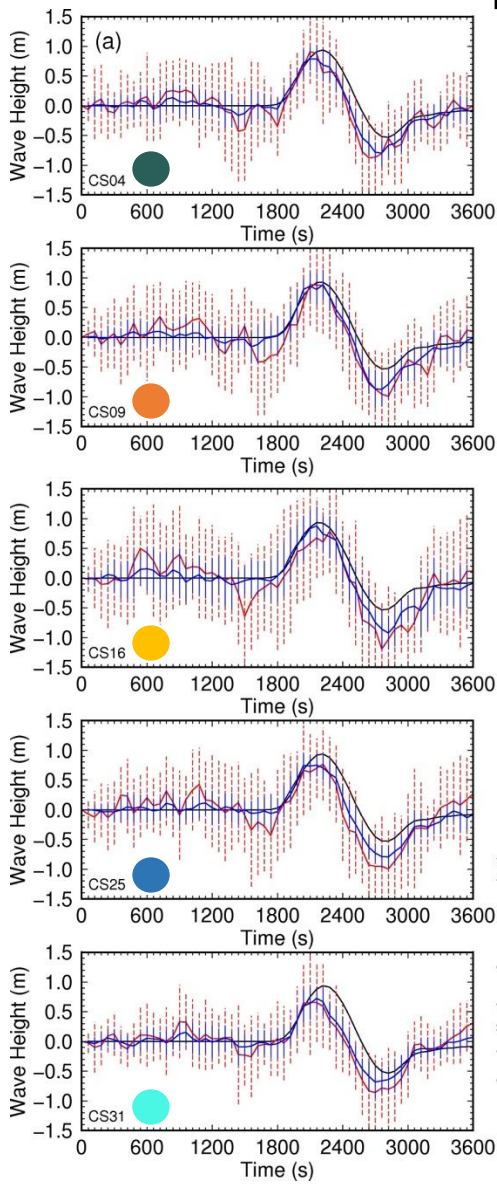

29%精度が上がる

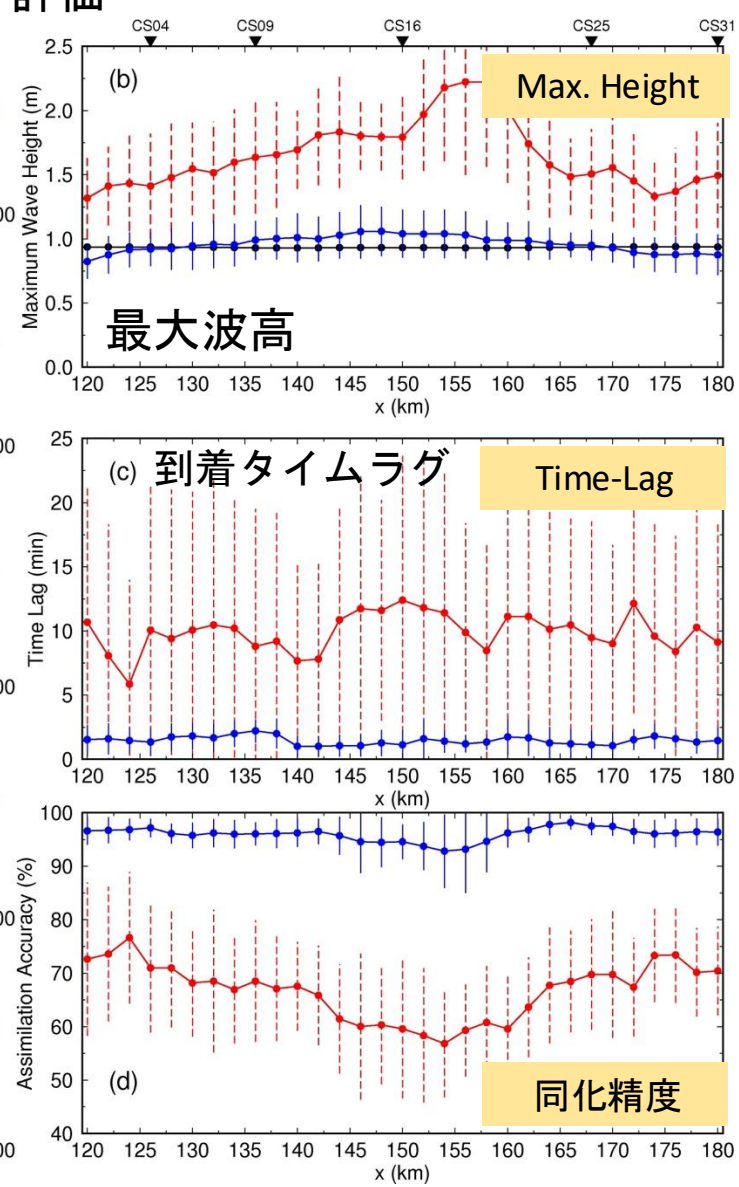
精度: 96%, 標準偏差: 1.3%

精度: 67%, 標準偏差: 5.3%

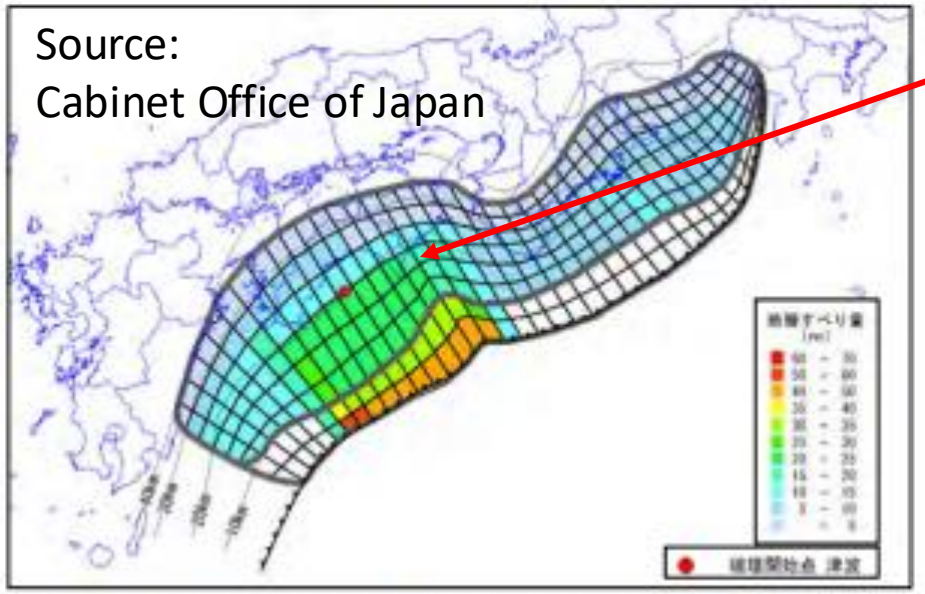
(Sahana et al., 2024)

## 評価



# ② Nankai Trough Earthquake (60-80% Probability of Occurrence)




【ケース④「四国沖」に「大すべり域＋超大すべり域」を設定】

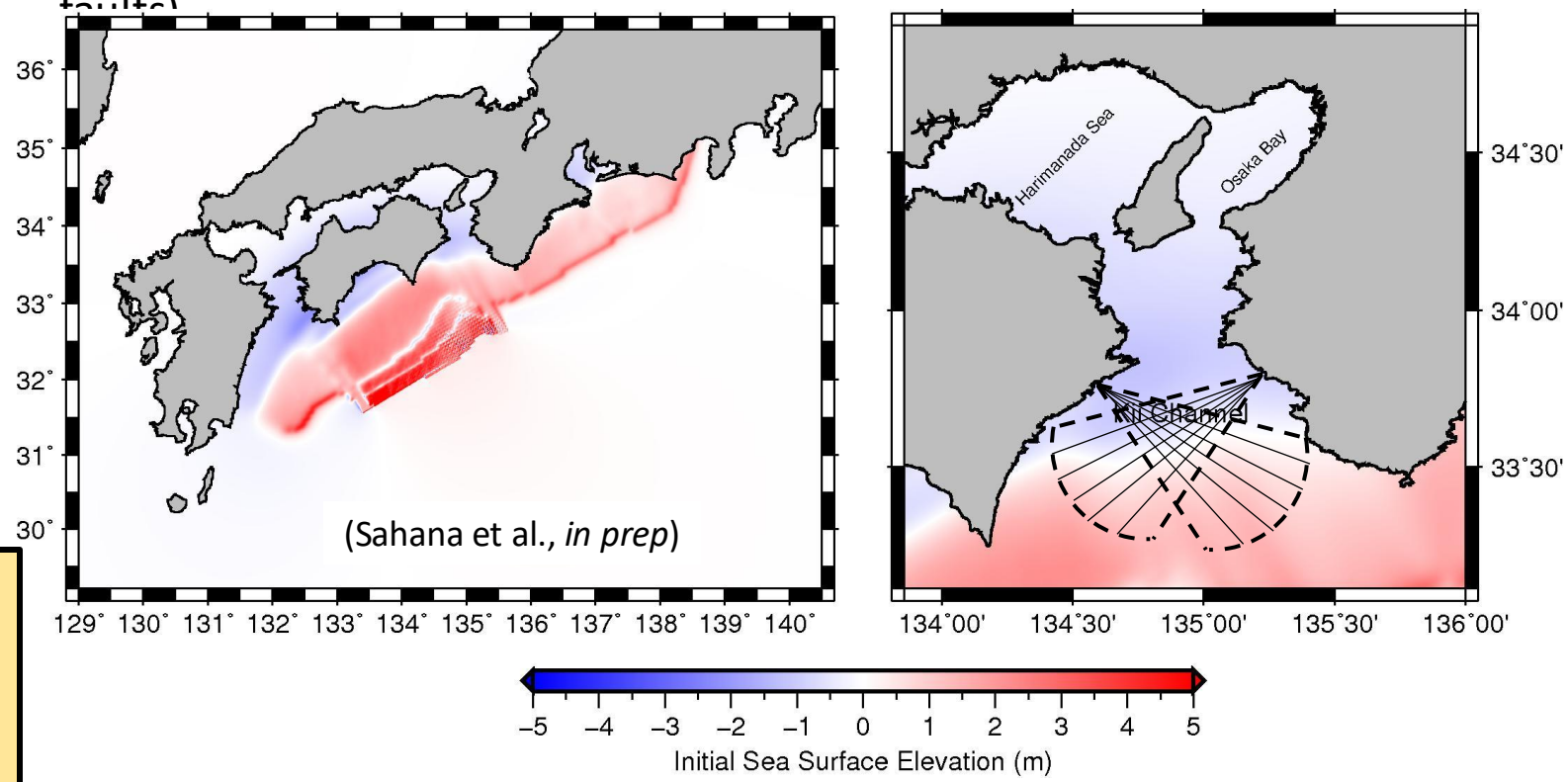
Source 震源: Shikoku

すべり額
断層の数 Number of Faults 5,773 個
最大地震すべり振幅 Maximum Slip Amplitude 60-70 m (50.6 m)
主な地震のすべり振幅 Dominant Slip Amplitude 25-30 m (14% of total faults)

Okada Formula
岡田式

JAGURS Program

断層パラメータによって生成される初期の海面高度を計算する式

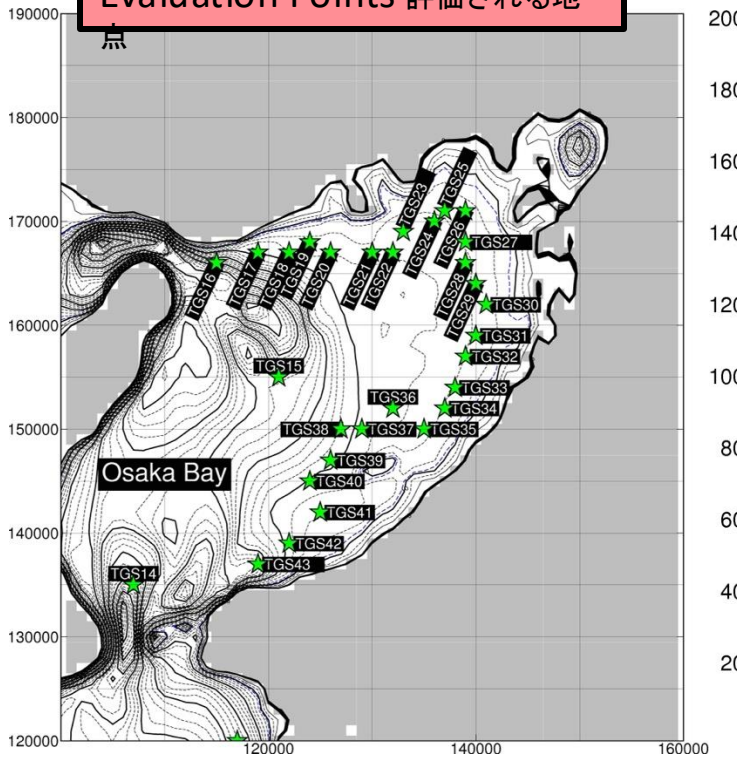



- ❖ Confirming the applicability of this study into real-world application (real bathymetry and tsunami scenario)
- ❖ 本研究の実際の応用性（実際の地形データと今後の津波シナリオ）および津波対策への影響の確認。

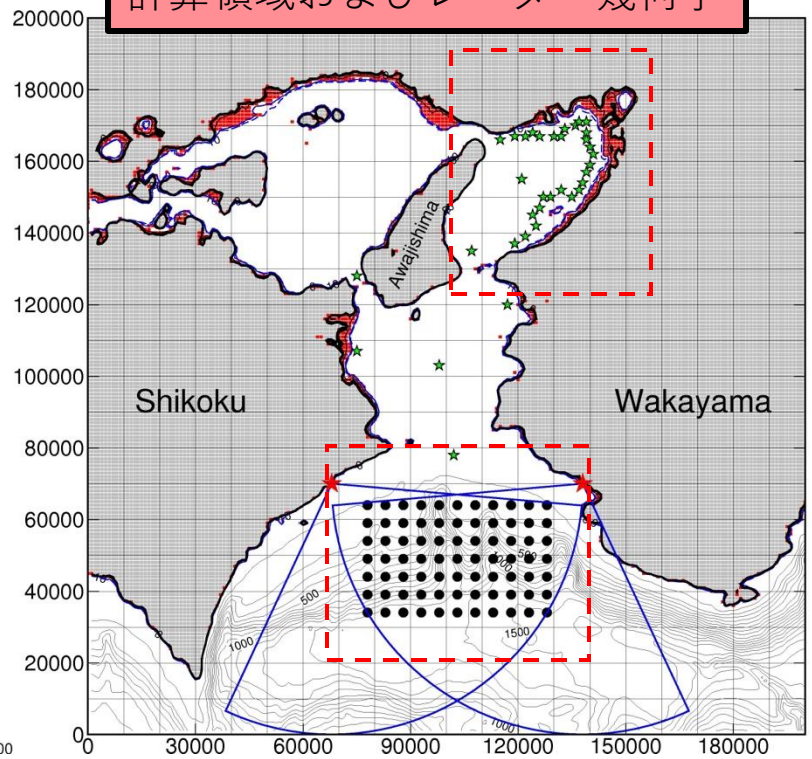
❖ Maximum SSE is exceeding **20 m** ❖ SSE inside Kii Channel ±**2 m** and $T_s$ is ~**60 min**

# ② Calculation Domain and Estimation of Measurement Error Distribution

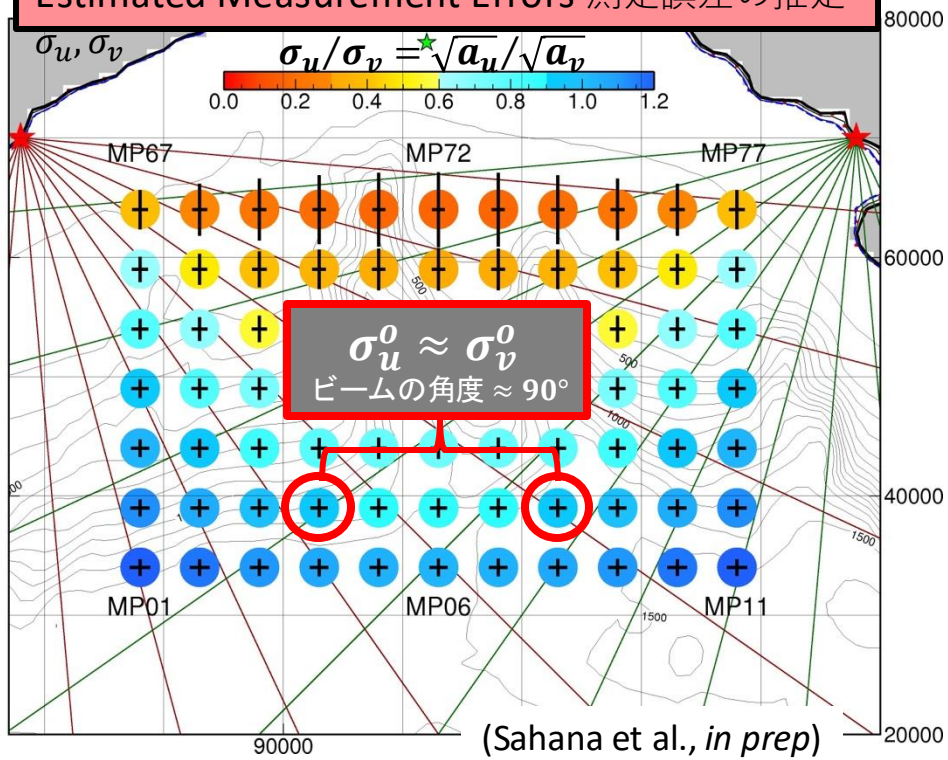
Evaluation Points 評価される地点




計算領域およびレーダー幾何学




Estimated Measurement Errors 測定誤差の推定




(Sahana et al., *in prep*)

About observation data $\boldsymbol{y_m} \rightarrow u_{obs}, v_{obs}$

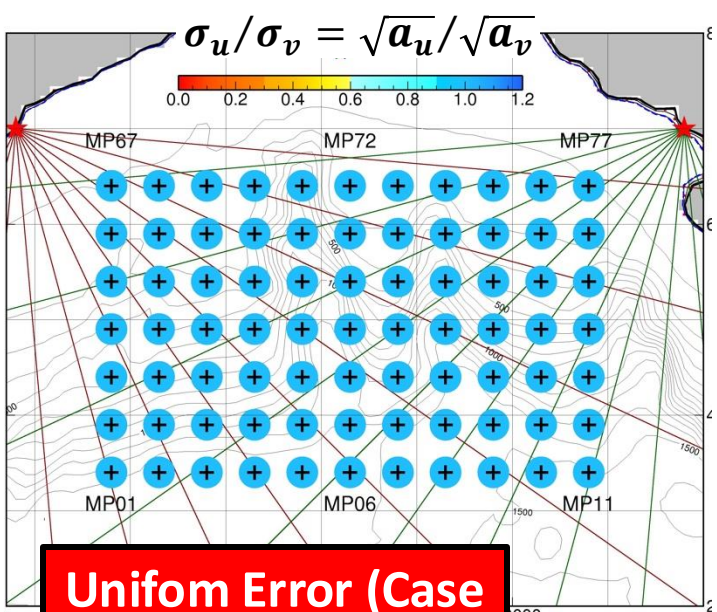
$$\boldsymbol{u_{obs,i}} = \boldsymbol{u_{sim.tsun,i}} + \boldsymbol{\sigma^o_{u,i}} = \boldsymbol{u_{sim.tsun}} + \boldsymbol{rand.}\left(\sqrt{a_{u,i}} \times \sigma_r^o\right)$$

$$\boldsymbol{v_{obs,i}} = \boldsymbol{v_{sim.tsun,i}} + \boldsymbol{\sigma^o_{v,i}} = \boldsymbol{v_{sim.tsun}} + \boldsymbol{rand.}\left(\sqrt{a_{v,i}} \times \sigma_r^o\right)$$

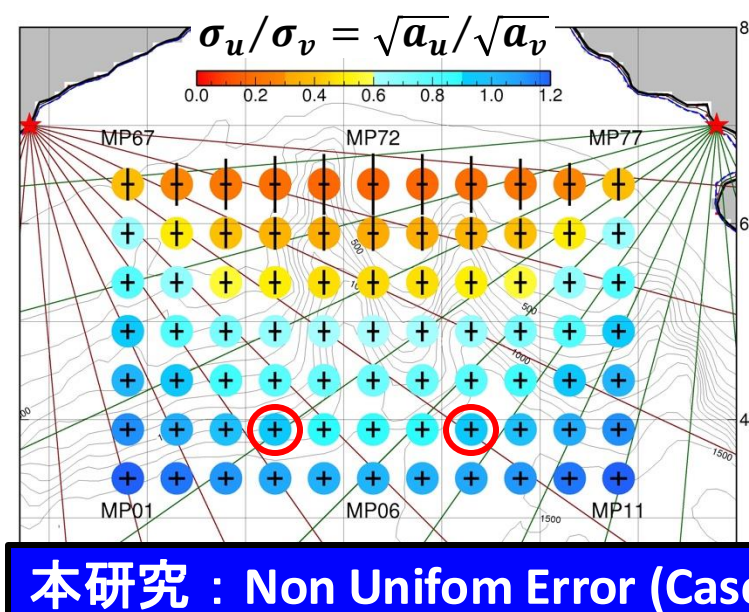
Where, $\boldsymbol{\sigma_r^o}$ = 5 cm/s

計算領域およびレーダー幾何学；測定誤差の推定

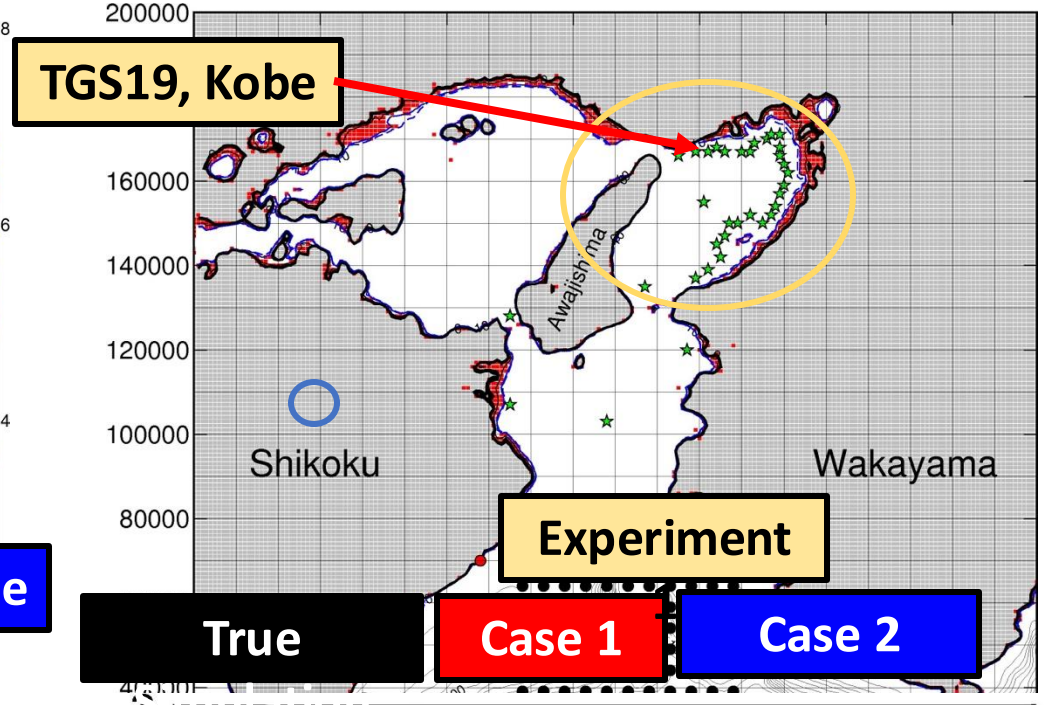
# Evaluation of Prediction Performance; Nonuniformity of Measurement Error




Unifom Error (Case




本研究：Non Unifom Error (Case




Amplitude of Tsunami

$$K = 1 - \frac{\left(H_f^{max} - H_s^{max}\right)^2 + \left(H_f^{arr} - H_s^{arr}\right)^2}{\left(H_s^{max}\right)^2 + \left(H_s^{arr}\right)^2}$$

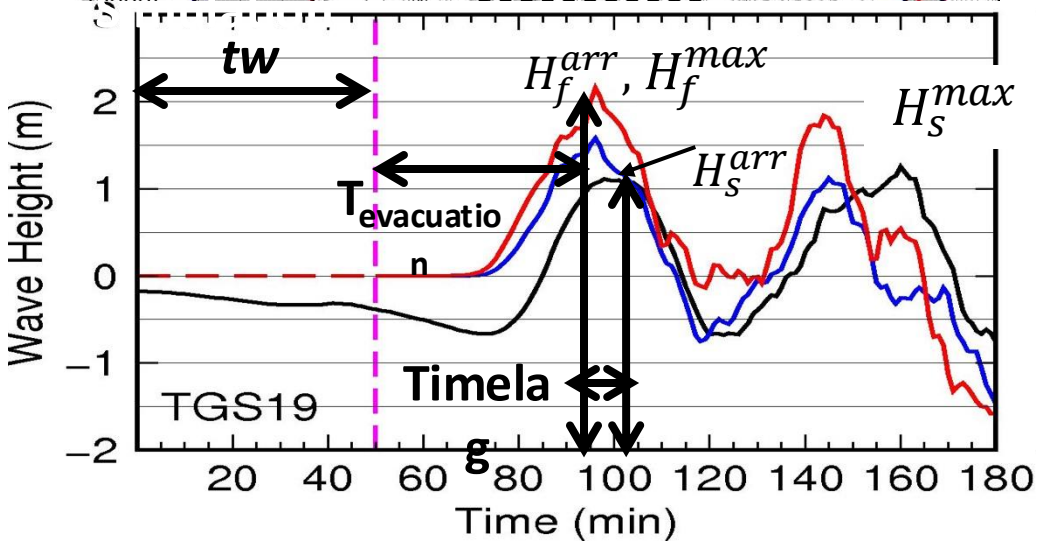
Time lag of Tsunami

$$L = \frac{\left|t_f^{arr} - t_s^{arr}\right|}{\left|t_f^{arr} - t_w\right|} = \frac{\left|t_f^{arr} - t_s^{arr}\right|}{T_{evacuation}}, where\ tw = 50\ \text{min}$$

スコアリング

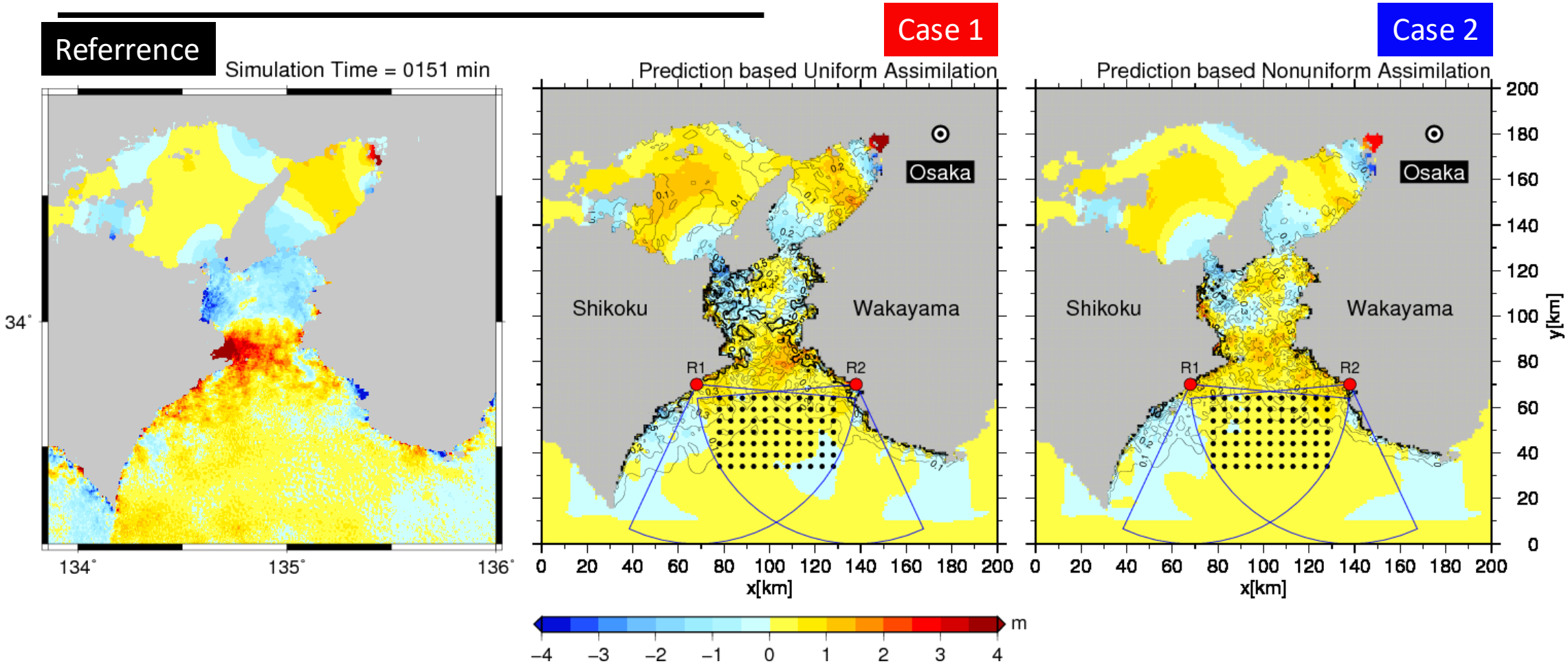
$$\boldsymbol{G = \alpha(1-K) + \beta L}$$

$$\boldsymbol{PA = (1-G) \times 100\%}$$




同化と予測性能の評価（スコアリング）

# Results: Assimilation and Prediction (Uniform and Non-Uniform Case) in NTS4-Multi

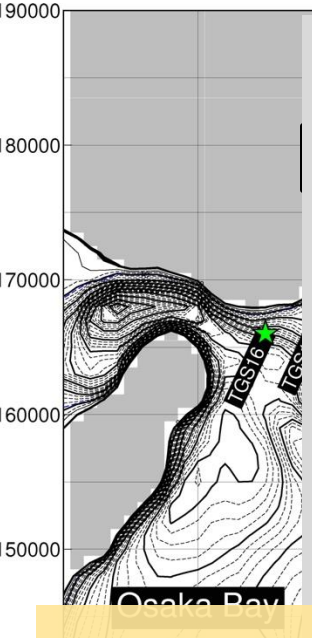


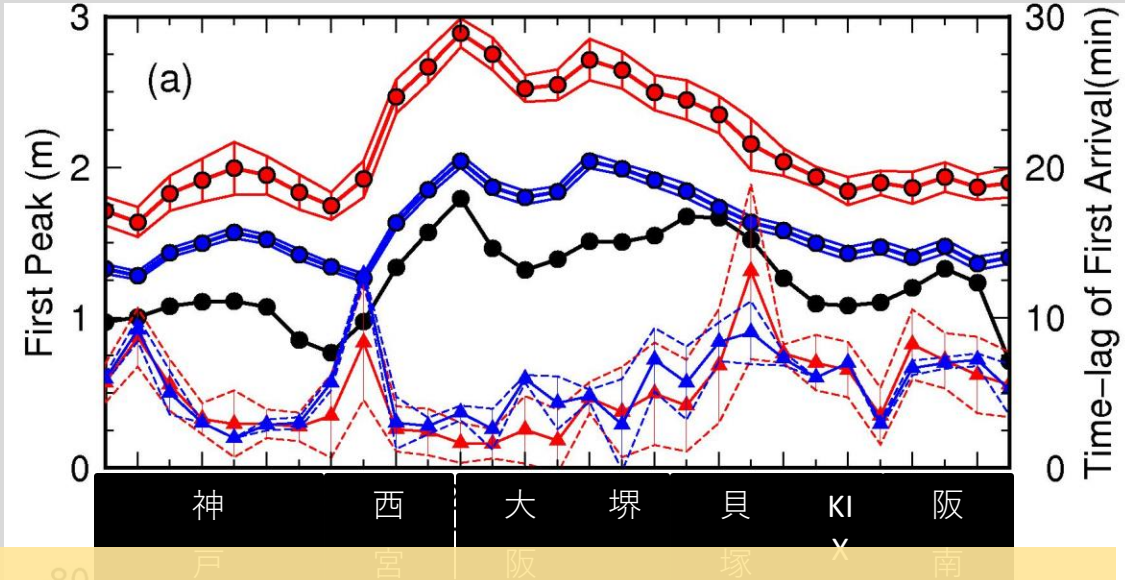

- Noisy wave height prediction is continuing.
- As the wave propagates to Harimanada Sea and Osaka Bay, STD of SSH decrease (dissipation energy filtered by two narrow straits)
- More stable prediction (low STD) was observed inside Kii Channel, Harimanada Sea, and Osaka Bay in Nonuniform case

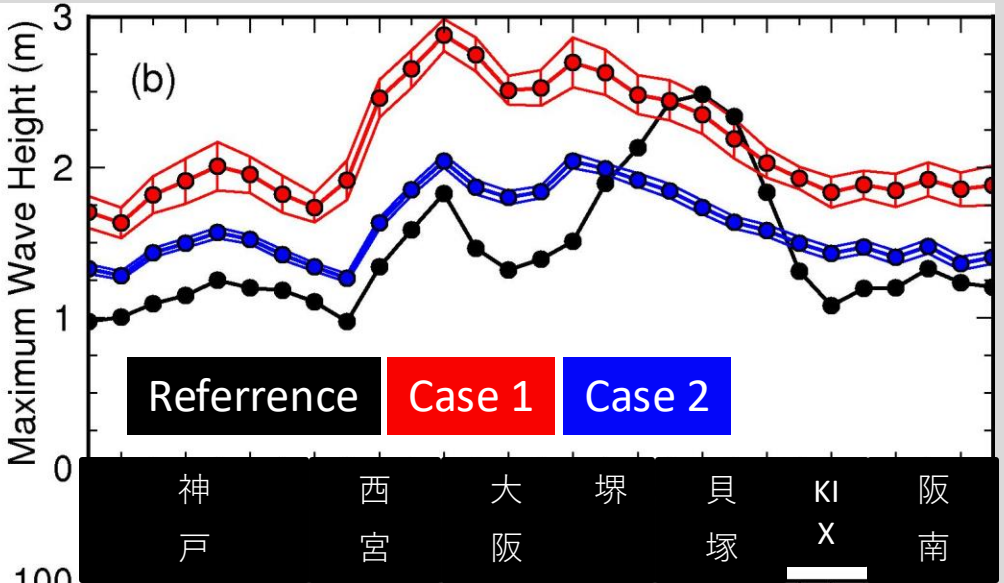
(Sahana et al., *in prep.*)

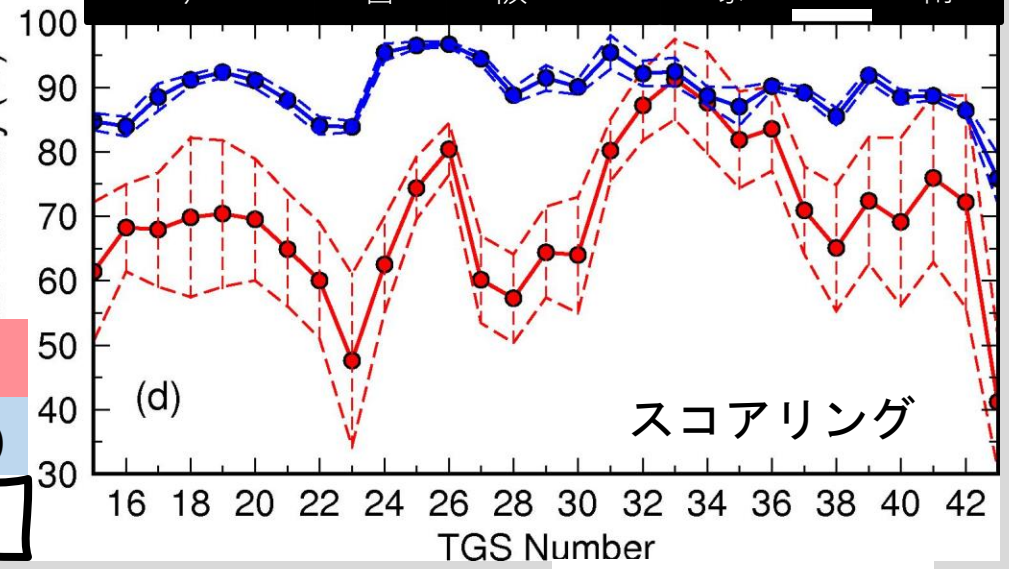
平均・標準偏差の同化と予測の津波流れの結果

# Results: Assimilation and Prediction (Uniform and Non-Uniform Case) in 15 independent experiments




**Consideration of $\sigma_u, \sigma_v$ distribrution** leads into

**a stable** assimilation performance with

**a low standard deviation** and **19% improvement in accuracy**.

Case 1 精度平均 $= \mathbf{70}\%\ (41-91\%)$, 標準偏差 $= \mathbf{8.7}\%\ (4-16.4\%)$

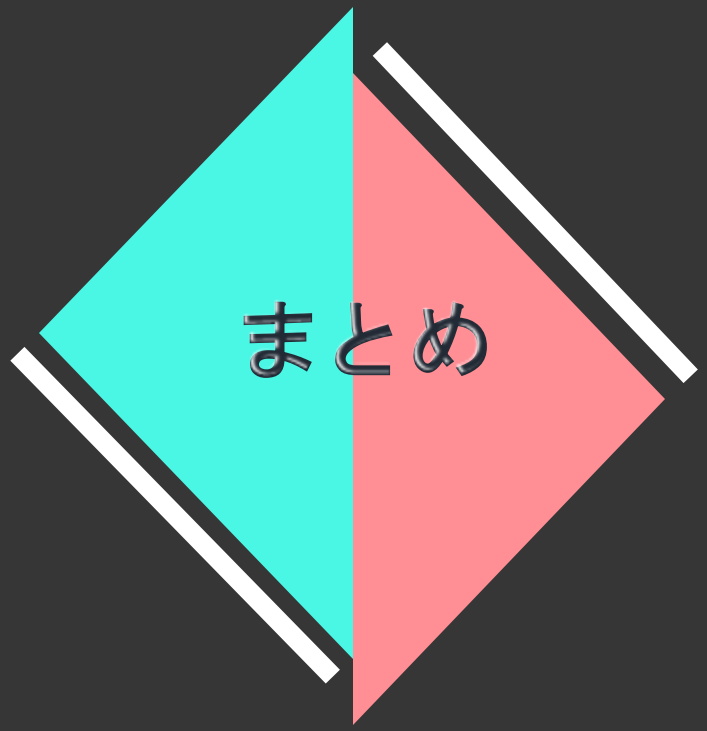
Case 2 精度平均 $= \mathbf{89}\%\ (76-97\%)$, 標準偏差 $= \mathbf{1.4}\%\ (0.4-3.8\%)$

**19 %** Improvement, and more **stable** （**本研究の結果**）

(Sahana et al., *in prep*)

平均・標準偏差の大阪湾における津波波高の時系列

# 紀伊水道における海洋レーダーを用いた津波データ同化への不均一誤差分布の適用




Summary:

1. Higher modes of EOF analysis were associated with the measurement errors of the velocity components.
   - ✓ Measurement errors in EW and NS directions were nonuniformly distributed, depended on crossing beam angle.

2. Incorporating beam-angle dependent measurement error distribution could improve the assimilation performance.
   - ✓ 19% improvement in accuracy across 15 times experiments with a Nankai Trough Scenario Case 4 and real bottom topography.